Abstract

The techniques based on fractals show promising results in the field of image understanding and visualization of high complexity data. In the aim to give an introduction to the theory of fractals the following topics will be summarised in this paper: the definition and analysis of fractals based on self-similarity and self-affinity behaviours, definitions for fractal dimension, fractal synthesis, multiresolution approach in the analysis and synthesis of fractals, fractals and hierarchic stochastic processes. The derived techniques with applications in geo-information processing and understanding will be underlined: generation of synthetic DEMs, fractal resampling of actual DEMs, algorithms for computation of the fractal dimension, unknown information modelling and data fusion, multiresolution synthesis and analysis of fractal images, multiresolution analysis and fractal dimension estimation. The paper presents also several experiments using fractals to generate accurate models for landforms and cover types, generation of synthetic images for model based picture processing, and image processing techniques for the analysis of remotely sensed images. The techniques have been applied both for optical and Synthetic Aperture Radar (SAR) image interpretation.
1 Introduction

The high complexity of remotely sensed images and measurements provided by the last generations of sensors demands new techniques for scene understanding and analysis. The similarity of fractal and real world objects was observed and intensively studied from the very beginning. The fractal geometry became a tool for computer graphics and data visualization in the simulation of the real world. In order to perform visual analysis and comparisons between natural and synthetic scenes several techniques have been developed. After a period of qualitative experiments fractal geometry began to be used for objective and accurate purposes: modelling image formation processes, generation of geometrically and radiometrically accurate synthetic scenes and images, evaluation of the characteristics of the relief, determination of the surface roughness, analysis of textures. The recent progresses in the mathematical formulation of the behaviors of the 1/f processes, in utilization of wavelets to make evidence of scale dependent features of nonstationary processes and the developments in hierarchic stochastic processes open a new perspective for the processing and interpretation of a large class of nonstationary multidimensional signals.

2 Elements of fractal geometry

A discussion of the field of fractals obviously embraces an enormous field [1]. The primary concern of this chapter is thus with the presentation of the elementary ideas necessary to understand applications of fractal geometry in geo-information processing [2]. Fractal geometry deals with the behaviors of sets of points $S$, in the $n$-dimensional space $\mathbb{R}^n$.

$$S \subset \mathbb{R}^n \quad (1)$$

For the addressed applications $S$ is a curve, a surface or an image intensity field. Therefore $n$ is restricted to 1, 2 or 3. But several applications, as multispectral data analysis, ask for representation of data in a higher dimension space [3].

2.1 Self-similarity and self-affinity

Mandelbrot defined a fractal as a shape made of parts similar to the whole in some way [4]. The definition is qualitative but not ambiguous, as it looks at the first glance. The main behavior of a fractal is its self-similarity [5]. A set is called self-similar if it can be expressed as a union of sets, each of which is a reduced copy of the full set. More general a set is said self-affine if it can be decomposed into subsets that can be linearly mapped into the full set. If the linear mapping is a rotation, translation or isotropic dilatation the set is self-similar. The self-similar objects are particular cases of self-affine ones.

$$\{ \text{SELF} - \text{AFFINE} \} \supset \{ \text{SELF} - \text{SIMILAR} \} \quad (2)$$

A fractal object is self-similar or self-affine at any scale. If the similarity is not described by deterministic laws stochastic resemblance criteria can be found. Such an object is said to be statistical self-similar. The natural fractal objects are statistically self-similar. A statistically self-similar fractal is by definition isotropic.

To have a more precise, quantitative, description of the fractal behavior of a set, a measure and
a dimension are introduced [6]. The rigorous mathematical description is done by the Hausdorff’s measure and dimension [7,8].

2.2 Hausdorff dimension

Let $S$ be a set of points in the $n$ dimensional space $R^n$. The topological dimension of the space is $n$, $n$ is an integer. Choose also a real number $r$ inferior to $n$.

$$S \subset R^n, 0 < r < n$$

One consider further the cover $H_\delta^r$ of the set $S$ with sets $U_i$ of limited diameter $|U_i|$. The Hausdorff dimension is defined:

$$H_\delta^r(S) = \inf \left\{ \sum_{i} |U_i|^r / 0 < |U_i| < \delta \right\}$$

$$|U| = \sup \{ |x - y| / x, y \in U \}$$

The infimum is evaluated over all coverings of $S$ by a collection of sets with diameters at most $\delta$. The set $\{U_i\}$ is countable or finite. $H_\delta^r(S)$ increases as $\delta$ decreases to zero. Decreasing $\delta$ the restrictions on the allowable coverings of the set $S$ are increasing.

The $r$-dimensional Hausdorff measure $H^r(S)$ of the set $S$ is defined:

$$H^r(S) = \lim_{\delta \to 0} H_\delta^r(S)$$

To exemplify: if $S$ is a smooth curve, $U_i$ can be a linear stick of length $\delta$ and $H^1(S)$ is the length of the curve, if $S$ is a smooth surface, $U_i$ can be a disk of diameter $\delta$ and $H^2(S)$ is the area of the surface. The Hausdorff measure generalizes the definition of length, area, volume. $H_\delta^r(S)$ gives the volume of a set $S$ as measured with a ruler of $\delta$ units. A figure with finite length will have zero area, and a finite area will be covered by a curve of infinite length. Based on these observations and particular cases, two properties of the Hausdorff measure will be introduced. If the $r$-dimensional Hausdorff dimension of the set $S$ is higher than zero, than the $p$-dimensional Hausdorff measure of the set $S$ is bounded, than the $p$-dimensional Hausdorff measure of the set $S$ is zero, for $p$ greater than $r$.

The value of the parameter $r$ for which the $r$-dimensional Hausdorff measure of the set jumps from zero to infinite is said the Hausdorff dimension, $\dim_H$, of the set $S$.

$$H^r(S) > 0 \forall p < r, H^p(S) = \infty$$

$$\dim_H S = \sup \{ r / H^r(S) = \infty \}$$

$$H^r(S) < \infty \forall p > r, H^p(S) = 0$$

$$\dim_H S = \inf \{ r / H^r(S) = 0 \}$$

A set is said fractal if its Hausdorff dimension strictly exceeds its topological dimension.

$$\dim_H S > n$$

Numerical evaluation of Hausdorff dimension is difficult because of the necessity to evaluate
the infimum of the measure over all the coverings of the set of interest. That is the reason to look for another definition for the dimension of a set.

2.3 Minkowski dimension

Minkowski dimension [6] allows the evaluation of the fractal feature of a set. First the parallel set \( E_S(\delta) \) of the set \( S \) is introduced.

\[
E_S(\delta) = \{ x \in \mathbb{R}^n / d(x, S) \leq \delta \} \tag{10}
\]

The parallel set \( E_S(\delta) \) of the set \( S \) is the set including all the points of the space that are closer than a given constant \( \delta \) to the points of the set \( S \). The Minkowski dimension of the set \( S \), is:

\[
\kappa(S) = \lim_{\delta \to 0} \sup \{ n - \frac{\log V_n(E_S(\delta))}{\log(\delta)} \} \tag{11}
\]

\( V_n \) represents the volume in the \( n \) dimensional space \( \mathbb{R}^n \).

In contrast to the box counting dimension or the Hausdorff dimension, to compute Minkowski dimension one must not search for an optimal cover of the set \( S \).

In figure 1 is exemplified the parallel set (the Minkowski sausage) of a fractal curve. Computing the volume of the parallel sets for different values of the radius \( \delta \), and plotting these values in log-log coordinates, one will obtain a straight line if the set has fractal behavior. The Minkowski dimension is computed as the topological dimension of the space, \( n \) minus the slope of the straight line. The similarities in the structure of the set \( S \) are detected evaluating the volume of the associated parallel set for different scales defined by \( \delta \).

2.4 Box counting dimension

The box counting dimension allows the evaluation of the dimension of sets of points spread in an \( n \)-dimensional space and also gives possibilities for easy algorithmic implementation.
Given a set of points \( S \), in a \( n \)-dimensional space \( \mathbb{R}^n \), and \( N_\delta \) is the least number of sets of diameter at most \( \delta \) that cover \( S \), the box counting dimension, \( \text{dim}_B \), is defined as:

\[
\text{dim}_B S = \lim_\delta \to 0 \frac{\log N_\delta (s)}{-\log (\delta)} \quad (12)
\]

Depending on the geometry of the box and the modality to cover the set, several box counting dimensions can be defined: 1) the least number of closed balls of radius \( \delta \) that cover \( S \), 2) the least number of sets of diameter at most \( \delta \) that cover \( S \), 3) the least numbers of cubes of side \( \delta \) that cover \( S \), 4) the number of cubes of the lattice of side \( \delta \) that intersect \( S \), 5) the largest number of disjoint balls of radius \( \delta \) centered in \( S \) [8].

The equivalence of these definitions was proved. Also it was proved that these dimensions are inferior bounded by the Hausdorff dimension [7].

In figure 2 the fourth definition is exemplified. A rectangular mesh of constant \( \delta \) is overlapped on the erratic line. The 26 “cubes “of the lattice that intersect the set are marked in black.

Using the box counting dimension, for the evaluation of the properties of discrete data sets, as always happens in real cases, careful interpretation of the results is asked. Different definitions give dimensions with different properties and can have different values for the same set.

![Fig.2 Box counting dimension](image)

To relate the scale properties of a fractal to the box counting dimension’s definition follows an example. Consider a fractal self-similar contour \( f(x,y) \), \( f(\alpha x, \alpha y) \) is statistically similar to \( f(x,y) \); \( \alpha \) is the scaling factor. The number of boxes of dimension \( \delta_x, \delta_y \) necessary to cover the set represented by the points of \( f(x,y) \) is \( N \), and the number of boxes of dimension \( \alpha \delta_x, \alpha \delta_y \) required to cover the set is \( N_\alpha \). If the set is self-similar, as previously supposed, the ratio \( N_\alpha / N \) will be a constant. The logarithm of this constant is proportional to the fractal dimension of the set.

If a statistically self-affine fractal is considered it will be non-isotropic. At different scales \( f(x,y) \) will be statistically similar to \( f(\alpha x, \alpha^H y) \). \( H \) is the Hausdorff dimension, and \( \alpha \) is the scaling factor. The boxes must be scaled differently in x and y direction, with \( \alpha \) and respectively \( \alpha^H \). The ratio \( N_\alpha / N \) is also a constant proportional to fractal dimension of the set.
2.5 Other dimensions

Farther several definitions for the dimension of a set have been introduced. All these definitions, as the previous ones also, have a common goal: to make evidence of the self-similarity or self-affinity of the set. As an immediate consequence all definitions are based on a multi-scale evaluation of a certain measure. For example:

**P(m,L)** dimension [9], is the probability to have m points within a box of size L. The expected number of boxes to cover the set N(L) is:

\[
N(L) = \sum_{m=1}^{N} \frac{1}{m} P(m,L) \tag{13}
\]

and the regression of N versus L in log-log plot gives a straight line if the set is a fractal one. The slope of the line is the fractal dimension of the given set.

**Space scale filtering**, the self-similarity or self-affinity properties of sets are scale relative, that is why one can deal with the change of the scale of the set instead of the change of magnitude of the “stick” used for the estimation of the dimension. For evaluation of the fractal dimension of a signal, \( \phi \), a multiresolution approach is used [10]. The signal is smoothed using a bank of Gaussian filters having different variance \( \sigma \).

**Covering-blanket method**, is used for the estimation of the fractal dimension of contours, surfaces or image intensities [11]. The concept of covering-blanket is based on the analysis of a multiscale construction. For a surface, as an example, the upper, and lower bounding surfaces are to be generated. The covering-blanket is defined by the band of thickness \( 2\varepsilon \), created by the two secondary functions. The multiscale analysis will be done for different values of \( \varepsilon \).

The estimation of the fractal dimension involves taking the logarithm of the difference of the upper and lower bounding surfaces divided by the scale factor \( \varepsilon \), and fitting a line to it in a log-log plot, as function of scale.

**Power-spectrum method**, is based on the property of fractal functions to have a negative power-low shaped power-spectrum function [12]. The Fourier transform is used to derive the power-spectrum and a linear regression is used in log space to derive the fractal dimension.

**Wavelet transform of fractals**, Both fractals and wavelets, as main characteristic, allow the scale to be made explicit [13]. The wavelet transform \( \mathbf{Wf} \) of a function \( \mathbf{f(x)} \) is its decomposi-
tion on a orthogonal basis of functions. The basis of functions is generated from a parent function \( \psi \) using dilatations of factor \( a \), and translations with vector \( b \) [14].

\[
Wf(a, b) = \int_{-\infty}^{\infty} f(x) \sqrt{a} \psi \left( a \left( x - b \right) \right) dx \tag{15}
\]

The wavelet transform encodes patterns occurring at different scales in a uniform way. It means that considering a fractal and computing its wavelet transform one can derive the fractal dimension [15].

3 Fractals synthesis

3.1 Brownian process

To generate fractal objects several techniques have been developed: deterministic fractals are synthesised using iterative equations, cellular automata, or L-systems; stochastic fractals are obtained by Brownian process simulation, using 1/f filtering methods, random-midpoint displacement or modified L-systems. Natural landforms are well represented by fractals derived from the Brownian process description [16].

Consider, in the one-dimensional case, a random process \( X(t) \). If the Probability Density Function PDF, of the consecutive samples is Gaussian the process is called Brownian.

\[
X(t_{n+1}) - X(t_n) = x(t_n) \quad t_{n+1} - t_n = t
\]

\[
p(x, t) = \frac{1}{\sqrt{4\pi St}} \exp \left( -\frac{x^2}{4St} \right) \tag{16}
\]

The Brownian motion describes as Gaussian the displacement of a particle in one time interval [17]. The displacement is a independent variable. \( S \) is a constant: the diffusion coefficient, it models the “spread” of a particle trajectory in Brownian motion. To study the scale behaviors, the Brownian process will be sampled at intervals \( \Theta = kt \). The PDF of the consecutive samples difference, for the new process, will be derived.

\[
\Theta = kt; \zeta = t \sqrt{k} \quad p(\zeta, \Theta) = \frac{1}{\sqrt{k}} p(x, t) \tag{17}
\]

The newly derived PDF at the scale \( \Theta = kt \) is also Gaussian and differs by a constant from the original process PDF. The Brownian process is statistically self-affine. The affinity is in the statistic of the differences of consecutive samples at any scale. Wiener introduced a random function to describe the displacement of particles in Brownian motion. The difference of consecutive samples is extracted from a Gaussian distribution and is proportional with a power function of the sampling period [4].
Where \( grv \) is a Gaussian Random Variable, and \( H = 1/2 \). The process is self-affine.

Mandelbrot generalized the random function of Wiener and introduced the concept of fractional Brownian motion, changing the exponent \( H \) to be any real number in the interval \((0,1)\). The new random function was denominated \( B_H(t) \). The previously presented random functions give a base for the generation of fractals in one dimension or in a n-dimensional space. The random functions can be represented in n-dimensions simply by the substitution of time in a space n-tuple of coordinates [18]. Voss introduced the successive random addition algorithm [16]. In order to generate a fractional Brownian curve the variance of the increments of the position must be:

\[
V(t) = E \{ [X(t) - X(0)]^2 \} = |t|^{2H} \sigma_0^2
\]

In the first iteration the increments of the process are to be extracted from a Gaussian distribution of variance \( \sigma = 1 \). In the n-th iteration the displacements (midpoints) are interlaced between the previous step points, and are extracted from a Gaussian distribution of variance:

\[
\sigma_n^2 = \left( \frac{1}{2} \right)^{2H} \sigma_{n-1}^2
\]

### 3.2 Spectral method

The power spectral density \( S \), of a self-affine fractal is a negative power law shaped function [19].

\[
S(f) \approx 1/f^\beta
\]

The fractal dimension \( D \) is related to the \( \beta \) coefficient [16]:

\[
D = T + (3 - \beta) / 2
\]

\( T \) is the topological dimension. As a direct consequence of this property the Fourier transform is one of the main tools for the generation of fractals objects.

### 3.3 L-systems

The plants three dimensional structure is probably the most realistic modelled using L-systems [20]. A formal set of rules specify how the plants do develop themselves in different stages. It is important to develop algorithms which by means of a reduced set of parameters can control the variability of the synthetic vegetation. The L-system is constructed starting with a string
called axiom, and in the first step substitute every symbol of the string in accordance to a given set of rules. The process is repeated iteratively. It is essential to note that all the symbols in the string are changed simultaneously. This is a major difference compared with a formal language were the parser is applied sequentially. Applying the previous procedure the defined object grows preserving the same structure at larger scales. More general than plant modelling, the L-systems can describe almost any fractal object or at least their finite approximations. The simplest class of L-system, DOL-system is exemplified further:

\[
\text{rule} \quad a \rightarrow ab \quad b \rightarrow a \\
\text{evolution} \quad b \quad a \quad ab \quad aba \quad abaab \quad abaababa
\]  

The work of Lindermayer was oriented mainly to graphic representation and considering the evolution of the field this issue was continued. The theory of L-systems developed in several new techniques: bracketed L-systems, a graph theoretic trees using strings with brackets [21]; data base amplification [2], simulation of development of real plants; axial tree [23], notion which complements the graph-theoretical concept and makes it closer to natural vegetation models; context-sensitive L-systems, that models the possible interaction of component elements. The techniques for plant models generation was enhanced using combined methodology of L-systems and iterated function systems IFS [24].

4 Multiresolution analysis and synthesis of 1/f fractal processes

A large diversity of fractal processes can be defined. One of the most important class of random fractals is considered to be the 1/f processes [25, 26]. These processes model a huge spectrum of natural and man-made phenomena. The recently developed mathematical methods for the representation of 1/f processes increased their impact in applications as signal and image processing.

Several models describe the 1/f process. One class, exemplified by the fractional Brownian motion, is based upon a fractional integral formulation. Other models are in the category of infinite order ARMA processes class. Recently stochastic multiscale and scale recursive models have been developed [27, 28].

The power spectra of the 1/f processes is

\[
S(\omega) = \frac{2}{|\omega|^\gamma} \tag{24}
\]

for a given range of the “gamma” parameter. In terms of spectral analysis Eq. 24 is not integrable and actually does not represent a valid power spectrum [29]. The 1/f random processes are self-similar, (Eq. 25 and 26)
characterized by a long term correlation structure with polynomial decay that can not be represented by classical time series models. An example is the correlation function of the fractional Brownian motion:

\[ R(t, \tau) = \frac{\sigma_H^2}{2} \left( |t|^{2H} + |\tau|^{2H} - |t-\tau|^{2H} \right) \]  
\[ \sigma_H^2 = \Gamma (1 - 2H) \cos \left( \frac{\pi H}{\pi H} \right) \]  

The processes are generally nonstationary but stationary when passed through time invariant linear filters. The nonstationarity of the process is related to the time dependent analysis, and the self-similarity is related to the scale-dependent analysis [30].

In terms of frequency characterization a 1/f process when filtered by an ideal bandpass filter yields wide-sense stationary random process with finite variance and having a power spectrum as in Eq. 24, with

\[ \gamma = 2H - 1 \]

This gives the mathematical frame to justify and accept former frequency synthesis methods for fractals synthesis in the natural bandpass assumption: the available data length limits the knowledge at low frequencies and the sampling interval limits the access to details at high frequencies. Sample functions of fractional Brownian motion have fractal behavior for

\[ 0 \leq H \leq 1 \quad 1 \leq \gamma \leq 3 \]

being characterized by the Hausdorff-Besicovitch dimension \( D=2-H \) (for one dimensional case). A limitation of the fractional Brownian motion is that it does not provide models for 1/f processes with

\[ \gamma < 1 \quad \gamma > 3 \quad \gamma = 1 \]

The multiresolution approach considers a decomposition of the signal space in nested sequences of approximation spaces [31]. The wavelet based multiresolution is characterized to have similar resolution approximation in all time intervals and at all scales. Because 1/f processes simultaneously exhibit both statistical scale invariance and a particular time invariance behavior the wavelet transform constitutes a natural analysis and synthesis tool for these processes.

The importance of the wavelet transform in the field of nonstationary 1/f processes can be compared with the importance of the Fourier transform in the field of stationary process. The wavelet transform applied to the 1/f processes has the same role as a Karhunen-Loeve expansion [15].

As a generalization a nearly 1/f process was introduced via wavelet based synthesis [15,25].
The process \( x(t) \)

\[
x(t) = \sum_m \sum_n x_m^n \omega_n^m(t) \tag{29}
\]

\[
\frac{\sigma_L^2}{|\omega|^\gamma} \leq S(\omega) \leq \frac{\sigma_H^2}{|\omega|^\gamma} \tag{30}
\]

is obtained from a collection of mutually uncorrelated zero-mean random variable \( x \), of variance

\[
\text{var} \left( x_n^m \right) = \sigma^2 2^{-\gamma m} \tag{31}
\]

for any orthonormal wavelet basis with a given degree of regularity and \( S \) the time averaged spectrum.

In figure 3 the interrelations fractals - wavlets are shown.

Fig.3 Inter relations fractals - wavelets.

As a result referring to the analysis of \( 1/f \) processes it was shown that the orthonormal wavelet expansion of such a process produces coefficients weakly correlated in contrast with the strong correlation of the original process. The wavelet coefficients are wide sense stationary at each scale and an “across scale stationarity” is also verified. For a fractal process the ratio of the energy of the detail signal at different scales is a constant logarithmically related to the fractal dimension.

The wavelet transform defines a bridge in between the rich class of multiscale stochastic processes and the fractional Brownian process. An isotropic stochastic process is indexed by the nodes of an homogeneous tree. The homogeneity is related to the “across scale stationarity”. A horizontal level in the tree is associated to a fixed scale resolution. The multiscale stochastic processes are described by a scale recursive model. The model can be interpreted as a generalization of the midpoint displacement technique for generation of Brownian motion. The technique assumes a dyadic partition of the unit interval and adding samples extracted from a random number generator according to the joint probability distribution implied by the Brownian motion model.
The construction can be further modeled as a sequence of interpolations at scales power of 2. For a linear spline interpolation the process can be interpreted as a nonorthogonal multiscale approximation using the triangular “hat” functions which are the integral of the Haar wavelet. At any level in the multiresolution representation each state is a linear combination of its parents and plus an independent noise.

The last advances in the description of 1/f noise, hierarchic stochastic processes and multiresolution representation give a new perspective in the analysis and synthesis of a large class of nonstationary signals. The diagram in figure 4 presents the direction of possible further developments.

Fig. 4 Direction of possible further developments

5 Applications

5.1 Incomplete data simulation

To deal with incomplete data, in the image formation process, fractal geometry can be used when models are unknown. The proved self-similarity or self-affinity of the land-forms and land cover structures are used. In figure 5 the synthesis technique is shown.

Fig. 5 Incomplete data simulation
The lack of geometric data encoded in Digital Elevation Models (DEM) and radiometric information is supplemented using fractal objects. The data fusion from real satellite images with synthetic images creates a new approach in model based image understanding. The model is expressed in terms of the geometry and radiometry of the synthetic images. The concept of identity declaration by physical modelling is considered.

Many of the early applications of fractal geometry were involved in finding methods for the generation of pleasant visual aspect images for computer graphic representations. More recently realistic looking landscapes have been synthesised for flight simulators or relief visualization of other planets [1, 26], and more precise simulations of the landforms are derived in the aim to build models to be used in feature evaluation for further correlation with characteristics of the natural relief [32]. Geomorphology and soil science ask for terrain models with peculiar characteristics for other simulations: water erosion process, drainage basins topology, surface water flow, river’s course erosion, wind mass transport effects, deforestations, volcanic lava flow [33, 34]. One of the benefits in cartography from fractal simulation is the possibility to generate synthetic digital elevation models at a variety of scales and terrain roughness which can be used as test areas for the performance of the algorithms for digitizing simulated cartographic maps [35]. The techniques frequently used for the generation of synthetic DEMs are the “mid point displacement”, and the simulation of the 1/f noise. A Gaussian white noise is filtered using a 1/f^β shaped transfer function. The output signal is a self-affine fractal having the fractal dimension D = T + (3 - β)/2. T is the topologic dimension of the space [36]. Figure 6 shows two synthetic DEMs having different fractal dimension. The surfaces are presented as Lambertian surfaces illuminated from the S-E direction. The left side image, 6a, due to the fractal behavior, can be considered as any rescaled subwindow from the right side image, 6b. The surface is self-affine, in statistical sense. The “lakes” in the right side image are obtained as the intersection of a plane with the surface. The contours of the lakes are fractal lines having the fractal dimension D - 1 [37].

![Fig.6 Synthetic fractal DEMs presented as Lambertian surface illuminated from S-E.](image)

Available DEMs are generally limited in resolution. The resolution is given by the constant of the support grid for the height data. Applications as: relief visualization, image formation modelling for remote sensors, high resolution contour mapping, ask accurate DEMs. A higher
resolution can be obtained by a resampling process: to add new samples in a higher resolution grid. If functional resampling is used the resulting DEM has an unrealistic smooth aspect. It was experimentally proved that the natural relief has fractal behavior for a certain range of scales [33, 38]. The fractal resampling process uses this prior information: the similarity of landforms for several spatial scales. The fractal resampling is accomplished in two steps: the analysis of the real DEM for evaluation of the fractal dimension and local variance of the height field, and the fractal interpolation [39]. The fractal interpolation increases the resolution of the DEM in steps of 2. The statistical resemblance of the synthesised samples is obtained using the random addition method of Voss [16]. An example is presented in figure 7: an 100 m resolution DEM resampled to 50 m. The surface is presented as a Lambertian one, lighted from S-E. The visual appearance is more realistic but not only, other applications where the exactitude of the DEM is required in stochastic sense are of practical importance. The generation of synthetical images in the aim to enhance the performance of classification for remotely sensed images is one of the topics of interest [40]. The fractal resampling can not be applied down a given scale. Gravity and diffusion process or, vegetation cover broke the continuity in similarity, and other models must be applied [41].

![Image](image_url)

**Fig.7 DEM, a - 100m resolution, b - fractal resampled DEM, 50m resolution**

### 5.2 Multiresolution approach in image synthesis

Surface visualization is limited in accuracy for at least two reasons: the limited resolution of the geometrical description, and incomplete knowledge and imperfect simulation of the light scattering process. For surfaces having fractal behaviors any “facet” or interpolated representation means a cut in the similarity. A multiresolution approach is used to derive an accurate model [42]:

1. **At macro-scale** the scene is described by the knowledge of the surface geometry.

2. **A meso-scale** is introduced relative to the spatial resolution of the sensor. The pixel intensity is dependent on the local geometry, on the local roughness of surface at a resolution...
higher than the sensor’s geometrical resolution, and on the reflectance behaviors of the surface at next finer scale. Thus the sensed intensity is the result of a nonlinear spatial operator. This spatial operator models the image formation process of the specific sensor.

3. At micro-scale the facets of the surface are characterized by reflectance functions. They are generally obtained by statistical, experimental or on heuristic basis.

The realism and accuracy of the synthetic images is determined by the model of local light scattering. A general scattering function for unpolarized light is a function of the four variables, the Bidirectional Reflectance Function, BRF [43].

At micro-scale the BRF can be approximated from simple models of light scattering. The model is dependent on three physical parameters describing the surface appearance: diffuseness, specularity and interreflection characteristics.

At meso-scale the previously presented models and the sensor description are considered. The image formation process is simulated. The sensor is represented by a non-linear spatial operator. In the usual case the operator is a spatial convolution of the surface reflectance with the point spread function of the sensor followed by a non-linear transform.

If the sensor resolution is lower than the resolution of the surface, the intensity of one pixel is modelled by the integral of all scattered intensities of the microfacets weighted by the sensor’s spread point function. A virtual radiometric experiment can be carried out for a given sensor and micro-geometry of the surface. In figure 8a the imaging geometry for a rough surface is presented. Each microfacet is characterized by the surface optical attributes and its geometry, the local normal vector n. The sensor and source of light positions are specified respectively by the vectors v and I. An area of 32 x 32 microfacets was imaged from a nadir placed sensor with variable incident illumination. The resulting images are displayed in figure 8b. The last scene is a perspective view of the imaged surface (3D). Using the information from these images the albedo and BRF have been computed, and with prior knowledge of the sensor characteristics the accurate pixels intensities have been modelled.

The experiment explains the difference in radiometry of the images in figure 5. After a fractal resampling the local roughness increases and the BRF is modified. The experiment was applied for vegetation cover radiometric evaluation. A three dimensional plant model was developed using generalized L-systems [44].

To take into account the exact surface roughness and to calculate the actual scattering crossection the Kirchoff solution must be found. The validity of the Kirchoff solution was intensively studied [45]. In the solution of scattering from fractal surfaces, the wavelength is considered as yardstick. The solution is derived for relative space-scale to wavelength ratios [45]. The results find applicability to synthetic aperture radar (SAR) imagery of sea surface or rough terrain.
The intrinsic similarity of the fractal surfaces for an infinite range of scales makes an accurate visualization impossible. Applying the previously presented models, and using the knowledge of the surface geometry at mesoscale and the sensor model, complex and more accurate reflectance functions have been obtained. The virtual radiometric experiment was applied for fractal surfaces.

Figures 9 a, b, and c show the dependence of the sensed intensity on the roughness, $\gamma$, and on the incidence light angle, $\alpha$, for three different assumptions of the micro-scale facets appearance: diffuse (a), diffuse and specular (b), and diffuse, specular, and interreflections (c). Note the highest roughness is given for low values of $\gamma$ (highest fractal dimension).
In figure 10 fractal surfaces are shown in macro-scale representation with diffuse and specular appearance. Figure 10a is generated with the appearance of the microfacets described by the diagram in figure 9a, and image 10b with the reflectance function as shown in figure 9b.

5.3 Application in understanding the remotely sensed scenes by optical sensors

The data fused in synthetic images, and the modelling of incomplete knowledge using fractal objects, is further demonstrated in a remote sensing application. The aim is to segment the snow covered areas in rough mountainous regions. This results in a new method for the rejection of topographic influences in the radiometry.

The importance of topographic effects on the radiometric behavior of the remotely sensed imagery increases for at least two reasons: the higher spatial resolution in the latest generation of satellite sensors, and the extension on remote sensing applications to rough mountainous areas. The higher spatial resolution of aerial imagery makes the analysis more sensitive to local terrain roughness. Interpretation of mountainous regions, faces one with difficulties during classification due to shadow areas and diffuse and indirect secondary lighting. The existing methods for the alleviation of topographical effects are based on models for light scattering that are local, implying that the model does not take into account the spatial resolution of the sensor, and the DEM resolution.
The previously deduced results in the virtual radiometric experiments have been used as base for a physical model for multisensor image and data fusion inference. Digital Elevation Models (DEM) of adequate resolution are available. This makes attractive the idea of geometrical modelling of the satellite images. Model refers to the geometry, to the radiometry of the imaged scene, and the simulation of the image formation process, the sensor and illumination models [46].

To alleviate the influences of the topography in a satellite image of a rough mountain area, synthetic images have been generated. For this experiment a Landsat-TM scene, band 4 (figure 11), was used. The spatial resolution of the sensor is approximately 30 meter. The data and knowledge fused in the synthetic image are: the geometry of the terrain, the sensor model, the sun position and an illumination model, the multiresolution assumption in image formation. The geometry is described by a Digital Elevation Model known in a 10 meter rectangular grid. The sensor was modelled by a convolution operator. The sun elevation and azimuth at the date and time of image acquisition are used. The illumination model is defined for direct lighted and shaded areas. Parallel illumination, illumination by interreflections, and diffuse light are considered. At micro-scale, 10 meters resolution, the surface is assumed to be described by
diffuse, specular reflections and interreflections. At meso-scale, 30 meters resolution, the sensor model was used to deduce the pixel intensity.

In figure 11 are presented: a) the perspective view of the DEM, b) the synthetic image modeling the snow cover, this image can be interpreted as a visualization of the fused data, c) the Landsat-TM band 4 scene, and d) the segmentation of the snow cover in the previous image. The radiometry of the synthetic image models rough surfaces covered by snow. Due to the more complete model specification, the accuracy obtained in snow cover segmentation is superior to other methods [47]. The presented algorithm describes the image formation process for direct lighted, and shadow areas, using the multiresolution approach.

5.4 Applications in Synthetic Aperture Radar image processing

The rough structure of many natural surfaces is reflected in a corresponding roughness of the pixel intensity of the imaged scene. The image is textured. If the surface is a fractal one, its image (the set of pixel intensities) will be also a fractal having the same dimension [18]. The idea is to use fractal geometry, the fractal dimension, as a feature to characterize the textures. A fractal transform is defined: an image is mapped in an other image that has as pixel intensity the values of the fractal dimension derived for a moving window overlapped on the original picture. It is necessary to state here several observations: one can derive a large class of very different objects having the same fractal dimension, real structure have fractal behaviors for several ranges of the scale, and it could be possible that the similarity is respected only for a very low number of scales, the natural scenes when imaged are very often corrupted by strong noise, the sampling and quantification process destroy the scale invariant patterns.

Several algorithms have been derived to enhance the discriminatory power of the fractal transform: multiple resolution techniques [48], lacunarity [49], local fractal dimension [10], dendritic analysis [50].

The rough aspect of the Synthetic Aperture Radar (SAR) images rises difficult problems in scene segmentation. The presence of the speckle phenomenon affects the performance of the algorithms for texture classification. In the mean time the filters applied to reduce the speckle noise change the texture features. The fractal dimension seems to be a promising global parameter for the classifications of the SAR images [51, 52, 53]. In figure 12, a SAR (ERS-1) image and its fractal transform are presented. The fractal transform was locally evaluated using the P(m,L) definition of the dimension. The urban area is segmented. Taking into account the previous observations referring to the difficulties to interpret the values of the various dimensions, the fractal transform is generally used as a feature in connection with other parameters. Better result in the applications of the fractal transform are reported in the segmentation of the images obtained from optical sensors and in the classification of the terrain roughness and geological features [55,56,57].
The fractal random process models have been used synthesis [58] analysis and segmentation of clutter in high resolution polarimetric synthetic aperture radar (SAR). The fractal dimension of a preprocessed SAR image was evaluated using the power spectral scaling algorithm. The clusters have been separated by a Maximum Likelihood classifier based on a Gaussian assumption applied to the fractal transformed image [59].

5.5 Miscellaneous

The algorithms derived on the previously presented results exploit the property that the coefficient of a suitable wavelet transform can be modelled as sets of mutually independent random variables having a specific geometric scale to scale variance progression. The algorithms implemented in this assumption are performant and compositionally efficient. Results have been obtained in implementing whitening filter for 1/f processes, Bayes detection, Maximum Likelihood parameter estimation, and smoothing of 1/f processes corrupted by Gaussian stationary noise [60].

A fractal deconvolution method was implemented for the analysis of aeromagnetic data. Iterative deconvolution procedure are used to recover the fractal innovation from data [61].

Particular topics in signal and image processing are the interpretation of undersampled periodicities, sampling with unevenly spaced intervals and noisy sparse data. Such data sets are difficult to process using conventional methods. Fractal algorithms derived from the theory of dynamic systems and chaos provide a alternative for classical techniques [62]. Several of the experimented algorithms are: analysis of Poincare section, analysis of multidimensional phase space object using the Radius of Gyration Exponent (ROGE), the Artificial Insymetra- tion Patterns (AIP), the Lipuanov Spectra, the R/S analysis and the Maximum Entropy method. These methods do not ask any physical background of the analysed data set, but if such a foundation of the signal generator process exists, this is a strong premise to guarantee the robustness of the analysis. The techniques could be classified as “model based”.

Fig. 12 Fractal transform: urban area segmentation in SAR scene
Fractal signal coding is a field that gives promising results and could be a basis for a further perspective development for the representation of the geo-information. The premises are in the fractal similarity of natural scenes and in the necessity to store or transmit huge mass of data geosciences generate.

Fractal signal coding aims the identification of a fractal or a set of fractals that are a best fit for a given signal and are represented on fewer bits than the original data. The fractal coding can be a feature extraction method but the main goal is data compression [63]. The mathematical basis for the fractal signal compression is in the theory of Iterated Function Systems (IFS) and Collage Theorem. The principle of the algorithms is in the field of block coding and vector quantization. The implementations, due to the high computational complexity are looking for fast searching algorithms and recently for the implementation of Genetic Algorithms (GA).

6 Conclusions

The field of fractals developed as an interdisciplinary area between branches of mathematics and physics and found applications in different sciences and engineering fields. In geo-information interpretation the applications developed from simple verifications of the fractal behavior of natural land structures, simulations of artificial landscapes and classification based on the evaluation of the fractal dimension to advanced remotely sensed image analysis, scene understanding and accurate geometric and radiometric modelling of land and land cover structures.

The multiresolution signal analysis and synthesis, mainly based on the theory of wavelets created a new perspective in the understanding of the stochastic fractals. The advances in modelling nonstationary and self-similar 1/f processes opened a new direction in signal processing. The hierarchic representation of the stochastic processes come as an algorithmic support creating a bridge connecting the statistic and deterministic approaches in analysis and synthesis. The models provide efficient scale recursive techniques for statistical signal processing allowing a trade-off accuracy to complexity.

Referring the computational effort, fractal analysis generally asks high complexity algorithms. Both wavelets and hierarchic representation allows now the implementation of “fast” algorithms or “parallel” ones. As a consequence it is expected a development of new experiments and operational applications.

Another direction is developing on the assumption of the multiresolution approach applied to accurate image synthesis. The physical model based method is for the simulation of unknown data. Prior parametric radiation scattering models and fractal geometric assumptions are encapsulated in the model at a sub-pixel scale. The results obtained for the modelling of vegetation and snow cover are promising and the research is directed to model refinements and algorithmic improvements. The topic is in the frame of data fusion applied for remotely sensed image understanding.

Remotely sensed images archivation and transmission ask performant compression algorithms. Data compression benefit from the theory of fractals through signal and image representation using IFS. The field is looking for efficient algorithms and also for an appropriate
problem statement. The advances are in Genetic Algorithms applications for fractal inversion and integration of techniques from vector and block quantization.

Two topics in data fusion have been presented, the utilization of fractal geometry for the simulation of incomplete data, and the data and knowledge fusion in synthetic images. The introduced method is an extension of the physical model inference in data fusion. The scene identification is accomplished by a second level of fusion of the real and synthetic images. The multiresolution approach in image synthesis is used to accurately represent and visualize surfaces with fractal nature. The similarity of fractal and real world is used as a base for incomplete knowledge modelling, and the results are applied for remotely sensed image processing. Better results have been obtained in the alleviation of the radiometric effects induced by the topography in the imagery of rough terrain regions.

7 References

17. J. Feder, Fractals, Plenum, 1988
33 M. Goodchild, “The fractional Brownian process as a terrain simulation model”, Modelling and Simulation Conference, Pittsburgh, Pa. 1982
48 S. Peleg, Multiple resolution texture analysis and classification, IEEE Tr. PAMI, -6, No. 4, 1985
49 J. M. Keller, S. Chen, Texture Description and Segmentation through Fractal Geometry, CVGIP, 45,1989.
53 A. Schistad, A. K. Jain, Texture analysis in the presence of speckle noise, IGRASS ’92