Theoretical study to the sub-project
“Interactive software for 2D and 3D standardization of pelvic radiographs and CT-scans for accurate evaluation of hip joint morphology”
under CO-ME project 4

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Abstract

The goal of the sub-project is a method to compensate mal-orientation of the pelvis in radiographs. The mal-orientation is supposed to be assessed using its correlation to the image distance between two bony landmarks. In this study it is investigated if this goal is theoretically feasible. Using the principle of error propagation, the clinical standard deviation of the image distance is estimated considering the errors entering the image acquisition. It resulted that the assessment of the pelvic rotation around the longitudinal axis could be possible, with a precision of about ±2.0°. Concerning the rotation around the transversal axis, however, an error of about ±8.0° has to be expected making the assessment of the pelvic tilt impracticable. The responsible entering error was found to be the variability of the pelvic anatomy.

Keywords — Radiograph, pelvic mal-orientation, bony landmarks, tilt reconstruction, error propagation.

1 Short description of the sub-project

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Goal: A method for compensating mal-orientation of the pelvis in radiographs in order to perform accurate measurements of the human acetabulum. The angles of mal-orientation are supposed to be assessed using their correlation to the distance between the mid of the symphysis pubis sy and the mid of the sacrococcygeal joint scj in the radiograph (part A). In part B, the change of the projection of the acetabular rim caused by a mal-orientation is supposed to be corrected. This is planned to be realized by means of the reconstructed angles of mal-orientation and the pre-determined shapes of the rim in the image at these angles.

2 Theoretical feasibility of the sub-project’s part A

The aim of part A is the reconstruction of the (mal-)orientation of the pelvis with respect to the medio-lateral axis (pelvic tilt) and to the cranio-caudal axis (pelvic rotation). The tilt angle is supposed to be assessed by means of the overall distance |sy − scj| between sy and scj, the rotation angle by means of the distance (sy − scj)ml between these image points in medio-lateral direction. The reconstruction of the pelvic orientation using these image distances is only possible if their clinical standard deviations are small compared to their decrease or increase due to the tilt or the rotation. The standard deviations depend on the following entering errors: the error in positioning the patient in the x-ray unit,
Figure 1: Fig. a shows \((\text{sy} - \text{scj})_{\text{ml}}\) subject to the angle of pelvic rotation, Fig. b \(|\text{sy} - \text{scj}|\) subject to the angle of tilt measured in the radiographs of the cadaver pelves. The dashed lines indicate the female, the dotted lines the male pelves.

the variability of the film–tube distance, the error in localizing \text{sy} and \text{scj} in the image, and the variability of the pelvic anatomy. The effect of the last two of these entering errors on the measured distances can be observed by means of an experimental set-up such as described in Sect. 2.1. The one of the other entering errors, however, can hardly be estimated experimentally. Therefore, a theoretical analysis is required. By application of the principle of error propagation, the standard deviations of \((\text{sy} - \text{scj})_{\text{ml}}\) and of \(|\text{sy} - \text{scj}|\) due to the entering errors were determined (Sect. 2.3). At first, however, the previous experimental results were theoretically reproduced in order to cross-validate the theoretical considerations with real data (Sect. 2.2).

2.1 Previous experiments

Twenty cadaver pelves (10 of each gender) were roentgenized at different rotation and tilt angles. It was investigated how \((\text{sy} - \text{scj})_{\text{ml}}\) is correlated to the pelvic rotation and \(|\text{sy} - \text{scj}|\) to the tilt. The pelves were mounted on a holding device allowing the controlled change of the orientation. The tube–film distance \(f\) was set to 120 cm, and the most dorsal pelvic point had a distance of 2 cm to the film. The neutral orientation of the pelves was chosen so that \((\text{sy} - \text{scj})_{\text{ml}}\) was zero and \(|\text{sy} - \text{scj}|\) had its mean value of 47.3 mm in female and of 32.3 mm in male pelves [1]. The rotation angle was varied between \(-9^\circ\) and \(9^\circ\) and the tilt angle between \(-12^\circ\) and \(12^\circ\). The stepsize was \(3^\circ\). Simulating the clinical situation, the horizontal shift of the pelvis caused by the changed orientation was compensated by directing the central beam at each exposure to the same pelvic point. This point was chosen to be the midpoint between the upper border of the symphysis and a horizontal line connecting both anterior iliac spines, according to the clinical praxis described in [1]. The measured values of \((\text{sy} - \text{scj})_{\text{ml}}\) and \(|\text{sy} - \text{scj}|\) subject to the rotation and to the tilt, respectively, are plotted in Fig. 1.

The aim was to obtain a function relating the measured distances with the angles of orientation. The curves in Fig. 1 were hence approximated by the linear functions

\[
(\text{sy} - \text{scj})_{\text{ml}}^{f/m} = k_{\beta}^{f/m} \cdot \beta + t_{\beta}^{f/m} \quad \text{and} \quad |\text{sy} - \text{scj}|^{f/m} = k_{\alpha}^{f/m} \cdot \alpha + t_{\alpha}^{f/m}
\]

where \(\alpha\) is the angle of tilt, and \(\beta\) the one of the rotation. The index \(f/m\) stands for female/male. The gradient and the offset were determined by fitting in EXCEL the mean of the measured curves. They were found to be \(k_{\alpha}^f = 2.348\), \(t_{\alpha}^f = 47.3\) mm, \(k_{\beta}^f = 2.364\),
\[ t^i_\beta = 0.0 \text{ mm in the female and } k^m_\alpha = 2.163, \ t^m_\alpha = 3.3 \text{ mm, } k^m_\beta = 2.138, \ t^m_\beta = 0.0 \text{ mm in the male pelvis.} \]

**2.2 Theoretical reproduction of the experimental observations**

The theoretical reproduction of the experimental results required the 3D coordinates of the following points in each pelvis:

- The mid of the cranial end of the symphysis pubis \((\text{Sy})\)
- The mid of the sacro-coccygeal joint \((\text{SCJ})\)
- The left and the right spina iliaca \((\text{Spl} \text{ and } \text{Spr})\)
- Two points defining the tilt axis \((\text{ATl} \text{ and } \text{ATr})\)
- The position of the axis of pelvic rotation \((\text{AR})\)
- The sacrum, that means the most dorsal point of the pelvis \((\text{D})\)

The coordinates were measured in the pelves mounted on the holding device by means of the positioning stick of a surgical navigation system. According to its definition given in [1], the intersection \(C\) of the central beam with the pelvis was calculated using

\[
C = \frac{1}{2} \left( \text{Sy} + \text{Spl} + \frac{\left( \text{Sy} - \text{Spl} \right) \cdot \left( \text{Spr} - \text{Spl} \right)}{|\text{Spr} - \text{Spl}|^2} (\text{Spr} - \text{Spl}) \right). \tag{2}
\]

First, the measured coordinates needed to be transformed to obtain the representation of the pelvic points in the coordinate system of the x-ray system \((x\text{-axis from left to right, } y\text{-axis from cranial to caudal, and } z\text{-axis parallel to the central beam})\). The coordinates vectors were rotated so that the tilt axis was parallel to the \(x\)-axis, i.e. so that \(\frac{\text{ATr} - \text{ATl}}{|\text{ATr} - \text{ATl}|}\) was \((1,0,0)\). Subsequently, the angles of the neutral orientation in the experiments were reconstructed. The angle \(\beta_0\) of pelvic rotation was found by solving \(\text{sy}_x(\beta_0) = \text{scj}_x(\beta_0)\). As the pelves were at neutral orientation with respect to the ventral-dorsal axis, it could be assumed that \((\text{sy} - \text{scj})_m = \text{sy}_x - \text{scj}_x\). The tilt angle \(\alpha_0\) was found by solving numerically \(|\text{sy}(\alpha_0) - \text{scj}(\alpha_0)| = 47.3 \text{ mm in the female and } |\text{sy}(\alpha_0) - \text{scj}(\alpha_0)| = 32.3 \text{ mm in the male pelves.}

Having determined the angles \(\alpha_0\) and \(\beta_0\), the pelvic points were moved to the neutral orientation through multiplication by \(\mathbf{R}_x(\beta_0)\mathbf{R}_x(\alpha_0)\), where \(\mathbf{R}_x\) indicates the matrix of rotation around the \(x\)-axis and \(\mathbf{R}_y\) the one of rotation around the \(y\)-axis.

The experimental pelvic rotation and the tilt were simulated by applying \(\mathbf{R}_y(\beta)\ (\beta = -9^\circ, -6^\circ, -3^\circ, ..., 9^\circ)\) and \(\mathbf{R}_x(\alpha)\ (\alpha = -12^\circ, -9^\circ, -6^\circ, ..., 12^\circ)\), respectively, to the pelvic points in the neutral orientation. Indicating these points with the index 0, their image coordinates are given by:

\[
sy_{x,y}(\alpha) = \frac{f \cdot \left( \mathbf{R}_x(\alpha)(\text{Sy}_0 - \mathbf{C}_0) \right)_{x,y} }{f \cdot \left( \mathbf{R}_x(\alpha)(\text{Sy}_0 - \text{ATl}) \right)_{x,y} + \text{ATl}z + f - 20 \text{ mm} - D_{0z}}
\]

\[
sy_{x,y}(\beta) = \frac{f \cdot \left( \mathbf{R}_y(\beta)(\text{Sy}_0 - \mathbf{C}_0) \right)_{x,y} }{f \cdot \left( \mathbf{R}_y(\beta)(\text{Sy}_0 - \text{AR}) \right)_{x,y} + \text{AR}z + f - 20 \text{ mm} - D_{0z}}
\]

\[
scj_{x,y}(\alpha) = \frac{f \cdot \left( \mathbf{R}_x(\alpha)(\text{SCJ}_0 - \mathbf{C}_0) \right)_{x,y} }{f \cdot \left( \mathbf{R}_x(\alpha)(\text{SCJ}_0 - \text{ATl}) \right)_{x,y} + \text{ATl}z + f - 20 \text{ mm} - D_{0z}}
\]

\[
scj_{x,y}(\beta) = \frac{f \cdot \left( \mathbf{R}_y(\beta)(\text{SCJ}_0 - \mathbf{C}_0) \right)_{x,y} }{f \cdot \left( \mathbf{R}_y(\beta)(\text{SCJ}_0 - \text{AR}) \right)_{x,y} + \text{AR}z + f - 20 \text{ mm} - D_{0z}}
\]

The subtraction of \(\mathbf{C}_0\) accommodates the fact that the intersection with the central beam
Figure 2: The experimental (black) and the theoretical (gray) mean values of $(sy_x - scj_x)$ (a) and $|sy - scj|$ (b). Again, the dashed lines indicate the female and the dotted lines the male pelves.

has per definition the $x$-$y$-coordinates $(0, 0)$. It leads to the cancellation of the rotational centers given by $AT_l$ and $AR$ in the numerator. The addition of $f - 20 mm - D_0$ is due to the fact that the $z$-coordinate of the most dorsal point is $f - 20 mm$ (see Sect.2.1). As the latter is only the case at the neutral orientation, $D_0$ is not multiplied by the rotation matrices and the rotational centers do not disappear in the denominator. The distances $sy_x(\beta) - scj_x(\beta)$ and $|sy(\alpha) - scj(\alpha)|$ were calculated inserting the coordinates as given by expr. (3). In Fig. 2, the mean of the calculated distances and the mean of the experimentally determined ones are plotted. Like in the evaluation of the experimental data, the mean curves were fitted by a linear function yielding $k_f^{\alpha} = 2.222$, $k_m^{\alpha} = 2.041$, $t_f^{\alpha} = 47.3 mm$, $t_m^{\alpha} = 3.2 mm$, $k_f^{\beta} = 2.233$, $k_m^{\beta} = 2.053$, $t_f^{\beta} = 0.1 mm$, $t_m^{\beta} = 0.0 mm$. The gradients $k_{f/m}^{\alpha,\beta}$ differ slightly from the experimentally determined ones. The reason may be that at a rotation or tilt the appearance in the image of the symphysis and the sacrococcygeal joint change and a different point is hence marked. However, regarding the variability of the measured data (see Fig. 1), the deviation between experimental and theoretical results is negligible. Thus, the theoretical considerations can be assumed to be correct.

2.3 The clinical standard deviations of $|sy - scj|$ and $(sy - scj)_{ml}$

The clinical standard deviations of the measured distances subject to the errors entering the acquisition and the evaluation of the radiographs were calculated. The principle of error propagation was used. Analytical expressions for the distances subject to variables representing the entering errors were formulated. The partial derivatives of these expressions with respect to the variables were calculated. Summing up the squared products between the derivatives at the variables’ mean values (zero by definition of the analytical expressions) and the variables’ standard deviations (the magnitudes of the entering errors) yielded the standard deviations of $(sy - scj)_{ml}$ and of $|sy - scj|$.

2.3.1 The variables representing the entering errors

The error variables are indicated with the symbols

- $\delta\alpha, \delta\beta, \delta\gamma$: variability of the pelvic orientation with respect the medio-lateral, the cranio-caudal, and the ventral-dorsal axis, respectively.
- $\delta T_x, \delta T_y$: error in medio-lateral and in cranio-caudal direction in directing the central beam at the targeted point.
• \( \delta T_z \): variability of the pelvic \( z \)-position due to a variation of the patient’s corpulence and to a tilt or rotation of the pelvis.

• \( \delta f \): variability of the film–tube distance

• \( \delta s_{x,y}, \delta s_{scj,x,y} \): error in \( x \) and in \( y \)-direction in localizing \( s_y \) and \( scj \), respectively.

• \( \delta \text{Sy} = (\delta S_{y_x}, \delta S_{y_y}, \delta S_{y_z})^T \) and \( \delta \text{SCJ} = (\delta SCJ_x, \delta SCJ_y, \delta SCJ_z)^T \): variability from pelvis to pelvis of the 3D position of \( \text{Sy} \) and \( \text{SCJ} \), respectively.

### 2.3.2 Analytical expressions for the measured distances

The main point in setting-up analytical expressions for the measured distances is the representation of \( s_y \) and \( scj \) subject to the error variables. The 3D coordinates of these points subject to the variable pelvic position and orientation at exposure are described by

\[
S_{y_{x,y}}(\delta \alpha, \delta \beta, \delta \gamma, \delta T_{x,y}, \delta \text{Sy}) = \left( R \left( \text{Sy}_0 + \delta \text{Sy} - C_0 \right) \right)_{x,y} + \delta T_{x,y} \tag{4}
\]

\[
S_{y_{2}}(\delta \alpha, \delta \beta, \delta \gamma, \delta T_{x,z}, \delta f, \delta \text{Sy}, \delta \text{P}) = \left( R \left( \text{Sy}_0 + \delta \text{Sy} - \text{P} - \delta \text{P} \right) \right)_{z} + P_z + \delta P_z + \delta T_z + \delta f \tag{5}
\]

\[
SCJ_{x,y}(\delta \alpha, \delta \beta, \delta \gamma, \delta T_{x,y}, \delta \text{SCJ}) = \left( R \left( \text{SCJ}_0 + \delta \text{SCJ} - C_0 \right) \right)_{x,y} + \delta T_{x,y} \tag{6}
\]

\[
SCJ_{2}(\delta \alpha, \delta \beta, \delta \gamma, \delta T_{x,z}, \delta f, \delta \text{SCJ}, \delta \text{P}) = \left( R \left( \text{SCJ}_0 + \delta \text{SCJ} - \text{P} - \delta \text{P} \right) \right)_{z} + P_z + \delta P_z + \delta T_z + \delta f .
\]

The index 0 indicates the pelvic coordinates at the neutral position and orientation of the pelvis, which were found as described in the following section. The symbol \( \text{P} \) stands for the center of rotation of the pelvis between two exposures, and \( \delta \text{P} \) for its variability. The subtraction of \( C_0 \) is due to the fact that the central beam is always directed to this point, independently from the pelvic orientation. The addition of \( \delta f \) is necessary, because an alteration of the film–tube distance leads to a change of the \( z \)-coordinates of the same amount. The matrix \( R \) describes the pelvic rotation between two exposures and is defined by

\[
R = \begin{pmatrix}
\cos \delta \beta \cos \delta \gamma & \sin \delta \alpha \sin \delta \beta \cos \delta \gamma - \cos \delta \alpha \sin \delta \gamma & \cos \delta \alpha \sin \delta \beta \cos \delta \gamma + \sin \delta \alpha \sin \delta \gamma \\
\cos \delta \beta \sin \delta \gamma & \sin \delta \alpha \sin \delta \beta \sin \delta \gamma + \cos \delta \alpha \cos \delta \gamma & \cos \delta \alpha \sin \delta \beta \sin \delta \gamma - \sin \delta \alpha \cos \delta \gamma \\
-\sin \delta \beta & \sin \delta \alpha \cos \delta \beta & \cos \delta \alpha \cos \delta \beta
\end{pmatrix}
\]

The expression for the measured distances can now be obtained by subtracting the projection of \( \text{SCJ} \) on the plane of the radiographic film from the projection of \( \text{Sy} \). The medio-lateral distance is hence described by

\[
(s_y - scj)_{ml}(\delta \alpha, \delta T_{x,z}, \delta f, \delta \text{Sy}, \delta \text{SCJ}, \delta \text{P}, \delta s_{x,y}, \delta s_{scj,x}) =
\]

\[
(f + \delta f) \left( \frac{S_{yx}}{S_{yx}} - \frac{SCJ_{scj}}{SCJ_{xy}} \right) + \delta s_{x,y} - \delta s_{scj,x} . \tag{7}
\]

where \( S_{y_{x,y}} \) and \( SCJ_{x,y} \) depend on the entering errors as described by (5). The variables \( \delta s_{x,y} \) and \( \delta s_{scj,x} \) are due to the error in localizing \( s_y \) and \( scj \). Actually, not the difference of the \( x \)-coordinates of these points in the image, but their distance in medio-lateral direction relative to the pelvis is measured. The measurement supposed to be independent from the pelvic orientation with respect to the ventral-dorsal axis. This fact was taken into account by setting the variable \( \delta \gamma \) to zero. Also \( \delta \beta \) was set to zero, to filter out the influence of this angle on the standard deviation of \( (s_y - scj)_{ml} \). Only then, the comparison of the change of \( (s_y - scj)_{ml} \) subject to the pelvic rotation with the variability of this distance due to the other entering errors is possible. For the same reason, \( \delta \alpha \) was set to zero in the following expression describing the absolute distance between \( s_y \) and \( scj \):

\[
|s_y - scj|(\delta \beta, \delta \gamma, \delta T_{x,y,z}, \delta f, \delta \text{Sy}, \delta \text{SCJ}, \delta \text{P}, \delta s_{x,y}, \delta s_{scj,x,y}) =
\]

\[
\sqrt{\left( (f + \delta f) \left( \frac{S_{yx}}{S_{yx}} - \frac{SCJ_{scj}}{SCJ_{xy}} \right) + \delta s_{x,y} - \delta s_{scj,x} \right)^2 + \left( (f + \delta f) \left( \frac{S_{yx}}{S_{yx}} - \frac{SCJ_{scj}}{SCJ_{xy}} \right) + \delta s_{y} - \delta s_{scj,y} \right)^2} . \tag{8}
\]
### Table 1: Required coordinates of the pelvis.

| pelvic point | female pelves | | male pelves |
|--------------|---------------|---------------|
| Sy<sub>0</sub> | –0.9 | 37.4 | 962.6 | –0.0 | 35.8 | 970.6 |
| SCJ<sub>0</sub> | –0.8 | –0.7 | 1089.5 | –1.1 | 10.9 | 1086.8 |
| C<sub>0</sub> | 0.0 | 0.0 | 959.3 | 0.0 | 0.0 | 964.4 |
| P | –1.3 | –27.7 | 1100.0 | 0.3 | –17.8 | 1100.0 |

#### 2.3.3 Estimating the required pelvic coordinates

The distance between sy and scj depends on the coordinates of Sy<sub>0</sub>, SCJ<sub>0</sub>, C<sub>0</sub>, and P. The general mean and the standard deviation of these coordinates were estimated using the measurements in the cadaver pelves (see Sect.2.2). First, the representation of the points in the coordinate system of the x-ray unit needed to be found. This representation can be achieved, e.g., by a rotation \( R_z(\phi) \) of the pelves with \( \phi \) around the z-axis so that \( (R_z(\phi)Sy)_{\text{x}} = (R_z(\phi)SCJ)_{\text{x}} \), and a subsequent rotation \( R_w(\theta, \phi) \) with \( \theta \) around \( w = R_z(\phi)Sy - R_z(\phi)SCJ \) so that both the spinae iliacae have the same y- and z-coordinates.

Due to the error in measuring the coordinates and to the imperfect anatomical symmetry, it is not possible that both the conditions were at the same time exactly fulfilled. Hence, the angles \( \phi \) and \( \theta \) were estimated in each pelvis solving the minimization problem

\[
\min_{\phi, \theta} \left( (R_z(\phi)Sy - R_z(\phi)SCJ)^2_x + (R_w(\theta, \phi)R_z(\phi)Spr - R_w(\theta, \phi)R_z(\phi)Spl)^2_y + (R_w(\theta, \phi)R_z(\phi)Spr - R_w(\theta, \phi)R_z(\phi)Spl)^2_z \right). \tag{8}
\]

Subsequently, the pelves needed to be tilted to their clinical standard orientation with respect to the medio-lateral axis (the x-axis). The pelves were rotated around the x-axis so that they all have the same tilt angle. This angle was chosen to be defined by the condition \( Sy_y = SCJ_y \). Then, the mean values of the coordinates in the female and the male pelves were calculated. Tilting these “mean pelves” so that the image distance between symphysis and saccro-iliac joint had the experimentally determined mean lengths of 47.3 mm / 32.3 mm yielded the clinical standard tilt angles \( \alpha_0 = -16.7^\circ \) / \(-12.1^\circ \). The coordinates vectors of all the cadaver pelves were transformed to the standard orientation by multiplication by \( R_x(\alpha_0) \).

Finally, the pelves needed to be shifted to the standard position. The x-y-coordinates of the intersection with the central beam were subtracted from all points to center the pelves. The z-position was determined based on the fact that the sacrum, i.e. the most dorsal point of the pelves, has approximately always the same distance to the x-ray film. Assuming that this distance is 10 cm including the mattress, the pelves were shifted along the z-axis so that the z-coordinate of the sacrum was \( f = 100 \text{ mm} \).

The mean values of the pelvic coordinates at the standard orientation and position are shown in Tab. 1. Assuming that the rotational center lies at the most dorsal pelvic point, for P the coordinates of D<sub>0</sub> were taken. The mean x-coordinates of Sy<sub>0</sub> and SCJ<sub>0</sub> differed from their expected value, i.e. zero, due to the error in measuring the points in the cadaver pelves. However, their deviation from zero of at maximum 1.1 mm has a negligible influence on the error estimation.
2.3.4 The magnitudes of the entering errors

The following estimates of the magnitudes of the entering errors were taken as the standard deviations $\sigma_{\delta\alpha}, \sigma_{\delta\beta}, \ldots$ of the variables:

- In [2], a mean change of pelvis orientation of $7.8^\circ$ with respect to the $x$-axis and of $4.5^\circ$ with respect to the $y$-axis is stated. Taking these angles as 66% confidence intervals, the corresponding standard deviations are $\sigma_{\delta\alpha} = 3.9^\circ$ and $\sigma_{\delta\beta} = 2.3^\circ$. The pelvis orientation with respect to the vetral-dorsal axis was estimated to be $\sigma_{\delta\gamma} = 3.0^\circ$.

- The error in directing the central beam towards the targeted point is estimated to be smaller in $x$- than in $y$-direction, because the patient's legs give a hint where the center line of the body is. It was assumed that $\sigma_{\delta T_x} = 10$mm and $\sigma_{\delta T_y} = 15$mm. The error components of the $z$-position resulting from the variable corpulence and the pelvic tilt and rotation are estimated to be $15$ mm and $20$ mm, respectively. Combining them yielded a total standard deviation of $\sigma_{\delta T_z} = \sqrt{15^2 + 20^2}$ mm = $25$ mm.

- $\sigma_{\delta f}$ was assumed to correspond to the scale division of the display of the film–tube distance, which is usually 10 mm.

- The precision in localizing $sy$ and $scj$ is limited by the pixel resolution of the x-ray image, by the alteration of the bony features due to a change of the pelvic orientation, and by the inter- and intra observer variability in selecting these points. Assuming that all these error components have the magnitude of one pixel, it is at the usual resolution of 150 dpi: $\sigma_{\delta sy_x} = \sigma_{\delta sy_y} = \sigma_{\delta scj_x} = \sigma_{\delta scj_y} = \sqrt{3} \cdot 0.16 = 0.3$ mm.

- For $\sigma_{\delta Sy}$ and $\sigma_{\delta SCJ}$, the standard deviations of the coordinates determined as described in Sect. 2.3.3 were taken. These standard deviations involve on the one hand the variability of the pelvic anatomy but on the other also the error in measuring the coordinates in the cadaver pelvis. Estimating the latter to be 2 mm, its square, 4 mm, was subtracted from the variances of the coordinates. This subtraction was omitted concerning the $x$-coordinate of the symphysis point. The measurement error plays no role at this point because of the choice of the neutral position and the definition of the intersection with the central beam (see previous section). The following standard deviations resulted in the female / the male pelves, respectively: $\sigma_{\delta Sy_x} = 0.11$ mm / 0.1 mm, $\sigma_{\delta Sy_y} = 4.3$ mm / 5.7 mm, $\sigma_{\delta Sy_z} = 11.1$ mm / 6.3 mm, $\sigma_{\delta SCJ_x} = 1.7$ mm / 1.4 mm, $\sigma_{\delta SCJ_y} = 4.1$ mm / 4.9 mm, $\sigma_{\delta SCJ_z} = 4.2$ mm / 3.3 mm.

- $\sigma_{\delta P_{x,y,z}}$ plays no role, as the partial derivatives of the measured distances with respect to $\delta P_{x,y,z}$ are zero at the mean values $\delta \alpha = \delta \beta = \delta \gamma = 0$.

2.3.5 Results

The following sums give the standard deviations $\sigma_{(sy-scj)_{ml}}^f$ and $\sigma_{(sy-scj)_{ml}}^m$ of the medio-lateral distance in female and in male pelves, respectively:

$$\sigma_{(sy-scj)_{ml}}^f = \sqrt{(0.15\sigma_{\delta T_x})^2 + (1.00\sigma_{\delta sy_x})^2 + (-1.00\sigma_{\delta scj_x})^2 + (1.25\sigma_{\delta Sy_y})^2 + (-1.10\sigma_{\delta SCJ_y})^2}$$

and

$$\sigma_{(sy-scj)_{ml}}^m = \sqrt{(0.13\sigma_{\delta T_x})^2 + (1.00\sigma_{\delta sy_x})^2 + (-1.00\sigma_{\delta scj_x})^2 + (1.24\sigma_{\delta Sy_y})^2 + (-1.10\sigma_{\delta SCJ_y})^2}$$ (9)
The standard deviations \( \sigma_{|sy-scj|}^f \) and \( \sigma_{|sy-scj|}^m \) of the overall distance are given by

\[
\sigma_{|sy-scj|}^f = (-0.04\sigma_{\delta\beta}^2) + (0.15\sigma_{\delta Ty}^2) + (-0.05\sigma_{\delta T_x}^2) + \\
(-0.01\sigma_{\delta\gamma}^2) + (0.02\sigma_{\delta Sy}^2) + (1.00\sigma_{\delta Sy_y}^2) + (-0.02\sigma_{\delta scj_y}^2) + \\
(-1.00\sigma_{\delta scj_x}^2) + (0.02\sigma_{\delta Sy_y}^2) + (1.25\sigma_{\delta Sy_y}^2) + (-0.05\sigma_{\delta Sy_y}^2) + \\
(-0.02\sigma_{\delta SCJ_x}^2) + (-1.10\sigma_{\delta SCJ_y}^2)
\]

and

\[
\sigma_{|sy-scj|}^m = (-0.08\sigma_{\delta\beta}^2) + (0.13\sigma_{\delta Ty}^2) + (-0.03\sigma_{\delta T_x}^2) + \\
(-0.01\sigma_{\delta\gamma}^2) + (0.04\sigma_{\delta Sy}^2) + (1.00\sigma_{\delta Sy_y}^2) + (-0.04\sigma_{\delta scj_y}^2) + \\
(-1.00\sigma_{\delta scj_x}^2) + (0.04\sigma_{\delta Sy_y}^2) + (1.24\sigma_{\delta Sy_y}^2) + (-0.05\sigma_{\delta Sy_y}^2) + \\
(-0.04\sigma_{\delta SCJ_x}^2) + (-1.10\sigma_{\delta SCJ_y}^2) + (0.01\sigma_{\delta SCJ_y}^2)
\]

Again, the upper expression concerns the female and the lower the male pelves. The coefficients in the above expressions are the partial derivatives of the measured distances with respect to the variables at \( \delta\alpha = \delta\beta = \delta\gamma = \ldots = 0 \). They are a measure for the influence of the entering errors. Regarding the partial derivatives and taking the magnitudes of the corresponding entering errors in the previous section into account, it became clear that \( (sy - scj)_m \) and \( |sy - scj| \) are mostly affected by the error in directing the central beam to the targeted pelvic point and by the variability of \( Sy \) and \( SCJ \).

Inserting the \( \sigma_{\delta\alpha}, \sigma_{\delta\beta}, \ldots \) in the expressions (10) and (11) yielded the standard deviations

\[
\sigma_{(sy-scj)_m}^f = 2.4 \text{ mm} \\
\sigma_{(sy-scj)_m}^m = 2.1 \text{ mm} \\
\sigma_{|sy-scj|}^f = 7.5 \text{ mm} \\
\sigma_{|sy-scj|}^m = 9.2 \text{ mm}.
\]

The great amount of the latter two standard deviations is mainly due to the great variability of \( Sy_y \) and \( SCJ_y \).

### 2.4 Discussion

#### 2.4.1 Clinical meaning of the found standard deviations

The plan of the investigated part A of the sub-project is to measure the distance \( (sy - scj)_m \) or \( |sy - scj| \) and to calculate the corresponding rotation or tilt angle \( \alpha' \) or \( \beta' \) using the following formula, which is derived from expression (1):

\[
\beta' = \left( (sy - scj)'_m - t_{\beta/m}^f \right) / k_{\beta/m}^f \\
\alpha' = \left( |sy - scj|' - t_{\alpha/m}^f \right) / k_{\alpha/m}^f
\]

The index ' indicates that the measured amounts of the distances are inserted in the above expressions. It is clinically important to know the probability that the pelvis is in fact orientated at the calculated angle. The probability \( P(\alpha \pm \Delta\alpha) \) or \( P(\beta \pm \Delta\beta) \) that the real angle lies in the range of \( \alpha \pm \Delta\alpha \) or \( \beta \pm \Delta\beta \) can be estimated using:

\[
P(\Delta\alpha) = \int_{\alpha' - \Delta\alpha}^{\alpha' + \Delta\alpha} \frac{k_{\alpha/m}^f}{\sigma_{|sy-scj|}_m} \cdot \sqrt{2\pi} \exp \left( - \frac{\left( |sy - scj|' - t_{\alpha/m}^f(\alpha) \right)^2}{\left( \sqrt{2 \sigma_{|sy-scj|}_m^2} \right)^2} \right) d\alpha
\]

\[
P(\Delta\beta) = \int_{\beta' - \Delta\beta}^{\beta' + \Delta\beta} \frac{k_{\beta/m}^f}{\sigma_{|sy-scj|}_m} \cdot \sqrt{2\pi} \exp \left( - \frac{\left( |sy - scj|'_m - (sy - scj)'_{m}(\beta) \right)^2}{\left( \sqrt{2 \sigma_{|sy-scj|}_m^2} \right)^2} \right) d\beta \quad (11)
\]

For \( |sy - scj|'_m(\alpha) \) and \( (sy - scj)'_{m}(\beta) \) the expressions (1) need to be inserted.
Figure 3: The medio-lateral distance \((sy - scj)_{ml}\) subject to the angles of pelvic rotation underlaid by the bounds of its theoretically estimated 68% and 95% confidence interval. The distances and the confidence intervals were calculated using the pelvic coordinates in the female (a) and in the male (b) pelves.

Figure 4: The distance \(|sy - scj|\) subject to the angles of pelvic tilt underlaid by the bounds of its theoretically estimated 68% and 95% confidence interval. The distances and the confidence intervals were calculated using the pelvic coordinates in the female (a) and in the male (b) pelves.
2.4.2 Conclusion and outlook

The targeted reconstruction of the angles of mal-orientation is assumed to be reliable if the probability that the real angle lies in a certain range close to the measured one is greater than 95%. Evaluating expression (11), it was found that this range must be $\Delta \beta = 2.1^\circ / 2.0^\circ$ and $\Delta \alpha = 6.6^\circ / 8.8^\circ$ in the female/male pelvises in order to obtain $P = 0.95$. This result can also be seen in the Figures 3 and 4, where the measured distances are plotted subject to the rotation and to the tilt angle, and the 68% and the 95% confidence intervals are underlaid. It means that the angle of rotation and of tilt can only be determined with a precision of about $\pm 2^\circ$ and $\pm 8^\circ$, respectively. Regarding these numbers, the reconstruction of the pelvic rotation can be assumed to be possible whereas the one of the tilt supposed to be limited to rare cases. In order to assure this conclusion, however, it is necessary to assess the degree of mal-orientation in the typical radiographs where a correction of the projection of the acetabular rim is planned.

Anyway, an important finding is that the confidence intervall of $|\mathbf{sy} - \mathbf{scj}|$ mainly depends on the variability of the cranio-caudal distance between $\mathbf{Sy}$ and $\mathbf{SCJ}$, i.e. the $y$-coordinates of these points. This variability was estimated using the data measured in cadaver pelvises. It depends on the orientation with respect to the medio-lateral axis of these pelvises. It has to be verified by means of clinical 3D data, if the orientation assumed here corresponds to the clinical one. Still, the project’s goal of reconstructing the pelvic tilt at exposure is considered feasible only at a reduced variability of $\mathbf{Sy}_y$ and $\mathbf{SCJ}_y$. This condition could be achieved, e.g., by assessing these coordinates individually in each patient using quantities which can be observed in the radiograph and which are correlated with the distance between $\mathbf{Sy}$ and $\mathbf{SCJ}$.

References
