RGB to Spectra

Nitre Challenge
Metamerism
Simple Image Formation

\[ \int_\omega Q(\lambda)E(\lambda)S(\lambda) \, d\lambda \]

\[ R(\lambda) = Q(\lambda)E(\lambda) \]

\[ \int_\omega R(\lambda)S(\lambda) \, d\lambda \]

Wandell, BA. “Foundations of Vision”
Observer Metamerism
Emissive chart for imager calibration

Jeffrey M. DiCarlo, Glen Eric Montgomery and Steven W. Trovinger
Hewlett-Packard Laboratories
Palo Alto, California

Figure 1. Responsivity functions for two hundred instances of a consumer camera plotted on top of one another. The widths of the lines indicate the responsivity variations across camera instances.
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Figure 1. Responsivity functions for two hundred instances of a consumer camera plotted on top of one another. The widths of the lines indicate the responsivity variations across camera instances.

Figure 6. The first prototype of the emissive calibration chart.

Figure 7. The spectral power distributions of the LED light sources used in the emissive calibration chart.
Illuminant Metamerism

Reflectance Curves of a Metameric Pair

Daylight

Fluorescent

Samples Appear To Match Under Daylight

Samples Do Not Match Under Fluorescent
Idea of Gamuts:
A stepping stone to thinking about metamers

Wire Frame: Adobe RGB
Solid: Epson Printer
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ii) This theoretical gamut is called the Object Colour Solid (OCS)

How the OCS is computed (efficiently) has been the source of research
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iii) How we solve for the OCS can help us solve for pairs of reflectances that are metamers
What is the colour response \( p \) in the direction \( n \) which has maximum length?

\[
\max_{S(\lambda)} \| n \cdot p \| \quad \text{s.t.} \quad \begin{cases} 
0 \leq S(\lambda) \leq 1 \\
p = \int_{\omega} R(\lambda) S(\lambda) d\lambda \\
(I_{3 \times 3} - nn^t)p = 0
\end{cases}
\]
What is the colour response $p$ in the direction $n$ which has maximum length?

In the discrete domain, this optimisation is a ‘Quadratic Program’. Can be solved efficiently. (Actually, can be further simplified as a linear program)
Object Colour Solid
Metamer Mismatch Volume

*a measure of how well spectrum is recovered*

\[ R(\lambda) \]

Many reflectances \( S(\lambda) \) integrate to the same ref RGB

\[ p_{\text{ref}} \]
Metamer Mismatch Volume

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\[
\overline{R}(\lambda) \quad \overline{Q}(\lambda)
\]

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\[
\max_{S(\lambda)} \| n \cdot q \| \quad \text{s.t.} \quad \begin{align*}
0 \leq S(\lambda) &\leq 1 \\
q &= \int_\omega Q(\lambda) S(\lambda) d\lambda
\end{align*}
\]
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\[
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**Metamer Mismatch Volume**

a measure of how well spectrum is recovered

---

**Diagram:**
--dessert-2.png
-dessert-3.png

\( R(\lambda) \)

\( \underline{p_{ref}} \)

\( \underline{Q(\lambda)} \)

\( B \quad G \quad R \quad B' \quad G' \quad R' \)
Metamer Mismatch Volume
a measure of how well spectrum is recovered

\[ R(\lambda) \]

\[ Q(\lambda) \]

Many reflectances \( S(\lambda) \) integrate to the same ref RGB

Solve for the ‘Mismatch volume’ analogous to how we solved for the OCS

\[
\max_{S(\lambda)} ||n \cdot q|| \quad s.t.
\]

\[
\begin{align*}
0 & \leq S(\lambda) \leq 1 \\
q & = \int_{\omega} Q(\lambda) S(\lambda) d\lambda \\
(I_{3 \times 3} - nn^t) q & = 0 \\
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\]
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Metamer Sets:
Can we recover ‘material’ from images?

And does this help solve vision tasks?
Discretely ...

\[ \int_{\omega} R(\lambda) S'(\lambda) d\lambda \]

\[ \rho = R^t S \]

3x1 RGB

Nx1 reflectance vector

Nx3 matrix of 'effective' sensitivities
The Set of all Reflectances

Hyper-cube constraint
(if 31 sample wavelengths then 31 dimensional hypercube)
Reflectances that ‘project’ to \( p \)

Reflectance hyper-plane constraint: All vectors \( S \) that ‘project’ to the same RGB lie on a plane
(if 31 sample wavelengths then 28 dimensional affine hyperplane or ‘flat’)

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\]
Reflectances that ‘project’ to $p$

Reflectance hyper-plane
constraint: All vectors $S$ that ‘project’ to the same RGB lie on a plane
(if 31 sample wavelengths then 28 dimensional affine hyperplane or ‘flat’)

$$\mathcal{R}^t S = p \equiv \mathcal{R}^t [\mathcal{R} \alpha + \mathcal{R}^\perp \beta] = p$$

$$S_{ref} + \sum_{i=1}^{28} Black_i(\lambda) \beta_i$$
Metamer Sets

The Metamer Set is the intersection of a hyper-cube with a hyper-plane (can have complex geometry [thousands of points, computationally hard])

\[ O(n^{d/2}) \]
Metamer Sets

The Metamer Set is the intersection of a hyper-cube with a hyper-plane (can have complex geometry [thousands of points, computationally hard]):

$$O(n^{d/2})$$

A hypercube is the intersection of half spaces delimited by the cube’s faces (hyperplanes).

Intersection of half spaces in N-D is found using ND convex hull algorithm.
\[
\int_{\omega} Q(\lambda) E(\lambda) S(\lambda) d\lambda = [47 \ 49 \ 53]
\]
Basic Metamer Sets

\[ \approx \]

denotes N-dimensional reflectances that ‘live’ inside the 31-d hypercube (N<<31)
Basis Approximations

\[ S(\lambda) \approx \sum_{i=1}^{N} \sigma_i S_i(\lambda) \]
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How many basis functions are needed?
Shown: ‘extremal’ metamers for grey reflectance assuming 7 dimensional CVA basis set

These are solved for *exactly* because

i) hypercube constraints in 7D are defined by 14 hyperplanes
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v) Can calculate basic metamer sets for 12, 13, 14 dimensional basis (more degrees of freedom than typically used)
Worked Example

\[ Q(\lambda) \]

Canon D1
Spectral Sensitivities

\[ E(\lambda) \]

\[ S(\lambda) \]
\[ \int Q(\lambda)E(\lambda)S(\lambda)\,d\lambda = [1.19, 4.24, 2.56] \]

The spectra shown lie on the convex hull of the ‘Metamer Set’

All spectra integrate to \([1.19, 4.24, 2.45]\)

All convex combinations integrate to \([1.19, 4.24, 2.45]\)
We are trying to characterise all reflectances which integrate to a single RGB.

\[
R^t S = p \equiv \\
R^t [\mathcal{R}_\alpha + \mathcal{R}^\perp \beta] = p
\]

\[
S_{ref} + \sum_{i=1}^{2} Black_i(\lambda) \beta_i
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The set comprises a ref spectrum that projects to the desired RGB and a combination of spectra that is orthogonal to the sensor space.

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If the reflectance basis is 5D, there is a single ref reflectance and a 2-dimensional set of metameric blacks.

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If the reflectance basis is 5D, there is a single ref reflectance and a 2-dimensional set of metameric blacks.

The metamer set is a 2-dimensional hyperplane in 5D (which in turn is embedded in 31-D).

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\]
Reprojecting

RGB

metamer set
Reprojecting

integrating (projecting) the metamer set to a second viewing condition typically generates multiple ‘colors’
Re-projecting to XYZ
Re-projecting to XYZ
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\[
\begin{pmatrix}
0.25 & 0.37 & 0.38 \\
0.24 & 0.36 & 0.38 \\
0.25 & 0.36 & 0.38 \\
0.25 & 0.35 & 0.38 \\
0.30 & 0.30 & 0.38 \\
0.31 & 0.32 & 0.38 \\
\end{pmatrix}
\]
Re-projecting to XYZ

\[ \begin{array}{ccc}
0.25 & 0.37 & 0.38 \\
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0.25 & 0.35 & 0.38 \\
0.30 & 0.30 & 0.38 \\
0.31 & 0.32 & 0.38 \\
\end{array} \]
8 dimensional reflectance basis yields 5 dimensional bounded convex region (in 8-d space) of metamers.

All of which integrate to \([1.19 \ 4.24 \ 2.56]\)
RGB to Spectra
How Many Dimensions?

Actual (blue) vs 3D approx

<table>
<thead>
<tr>
<th>% Reflectance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
</tr>
<tr>
<td>0.6</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.4</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
</tr>
</tbody>
</table>

400 450 500 550 600 650 700

ground-truth

closest in X-D metamer set
How Many Dimensions?

Actual (blue) vs 3D approx

Actual (blue) vs 6D approx

% Reflectance

Wavelength (Nanometres)

ground-truth

closest in X-D metamer set
How Many Dimensions?

Actual (blue) vs 3D approx
Actual (blue) vs 6D approx
Actual (blue) vs 9D approx

% Reflectance

Wavelength (Nanometres)

ground-truth
closest in X-D metamer set
How Many Dimensions?

Actual (blue) vs 3D approx

Actual (blue) vs 6D approx

Actual (blue) vs 9D approx

Actual (blue) vs 12D approx

% Reflectance

Wave

ground-truth

closest in X-D metamer set
Variants on a theme
Noise: Example

1. Well capacity 50000 electrons
2. Between .1 and 200 photons/10Nm wavelength band
3. 10 bit quantization
4. Sensors scaled so one sensor at one wavelength captures 100% of the photons
5. Shot-noise (assumed normally distributed.)
What effect does noise have on the metamer set?

Intuitively, it becomes larger
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normal metamer set

\[ R_k, S = p_k , \ k \in \{ R, G, B \} \]
\[ 0 \leq S \leq 1 \]
What effect does noise have on the metamer set?

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normal metamer set

$$\mathcal{R}_k S = p_k, \ k \in \{R, G, B\}$$

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with $\epsilon$ noise

$$p_k - \epsilon \leq \mathcal{R}_k S \leq p_k + \epsilon, \ k \in \{R, G, B\}$$

$$0 \leq S \leq 1$$
Noise

The ‘paramer’ set (computed with ~2% noise) is $3^*$ the volume of the noise free Metamer set.
Noise vs Dimensionality
60% grey that maps to a single RGB could be the ‘projection of many metamers.
Under a second light the metamer set spans a set of possible RGBs.
Invariant Metamer Sets
Metamer Set: 6 dimensional basis

Metamer Set: 10 dimensional basis
What are good sensors?

i) The projection in the ‘luminance’ direction has small metamer sets

ii) Possibly, given N>3 sensors, there are linear combinations which give RGB (sRGB?) with small metamer sets (use >3 sensors to get provably stable 3 sensor measurements)

iii) It can’t just be about metamers. Below is an sensor set that has many of the same metamer sets for all lights
Summary
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5) Next Steps? Metamer sets and other tools exist for answering a variety of questions. Example: “Metamer Constrained Colour Correction”