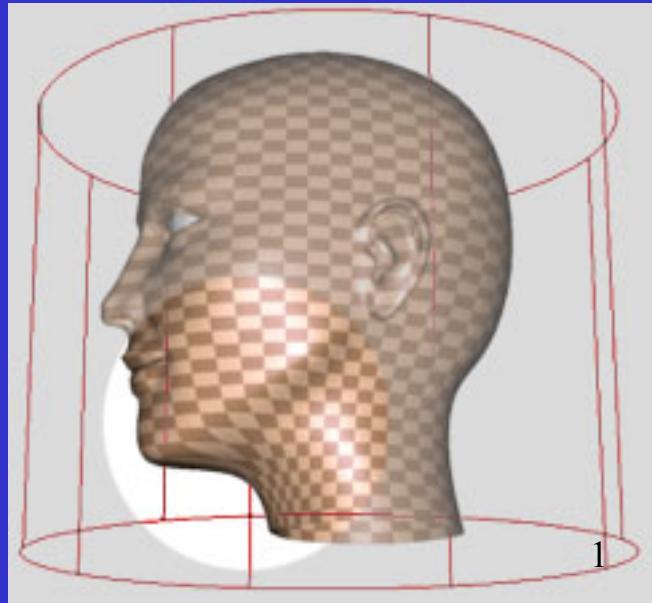
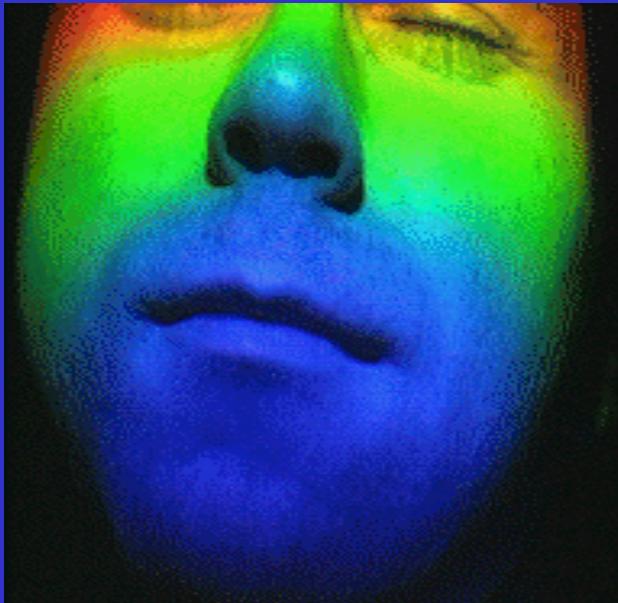


# Surface Features: colour and texture



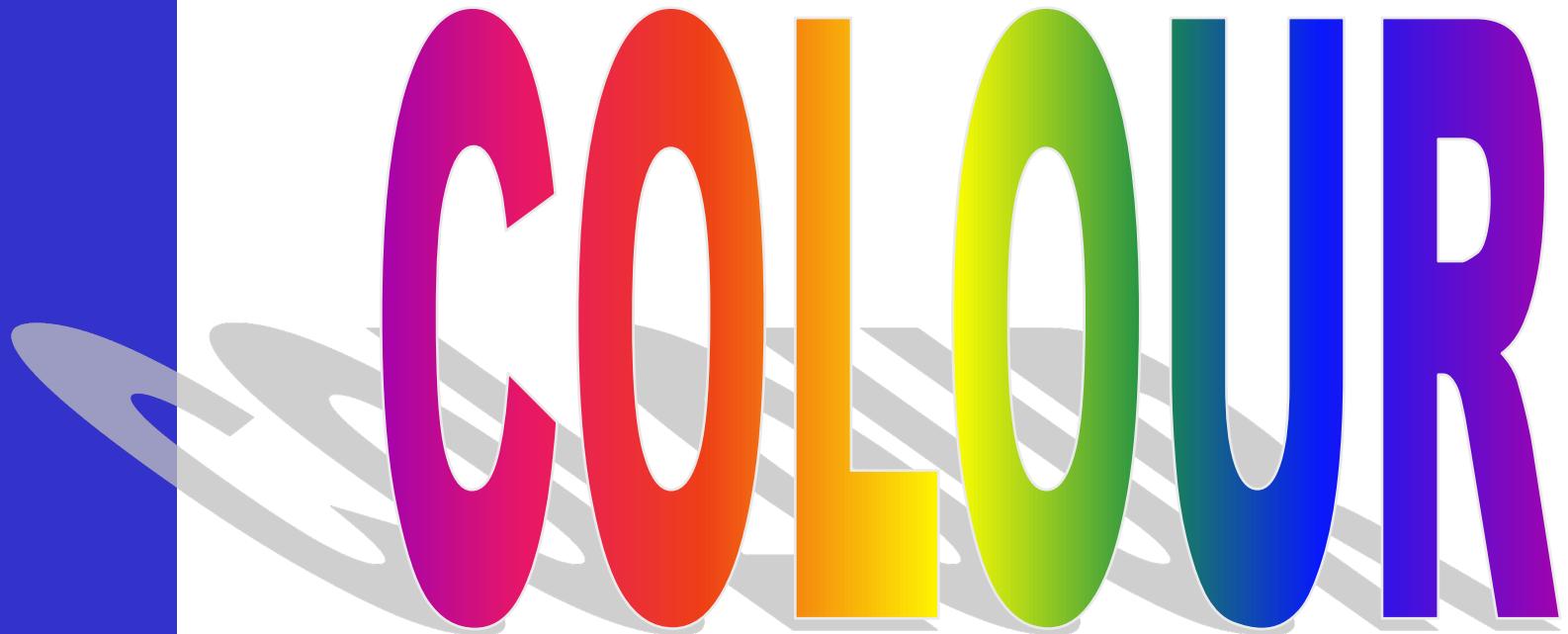
## Introduction

### *colour*

- color spaces
- colour constancy
- surface reflectance revisited
- illumination invariant colour features
- the holy grail: BRDFs

### *texture*

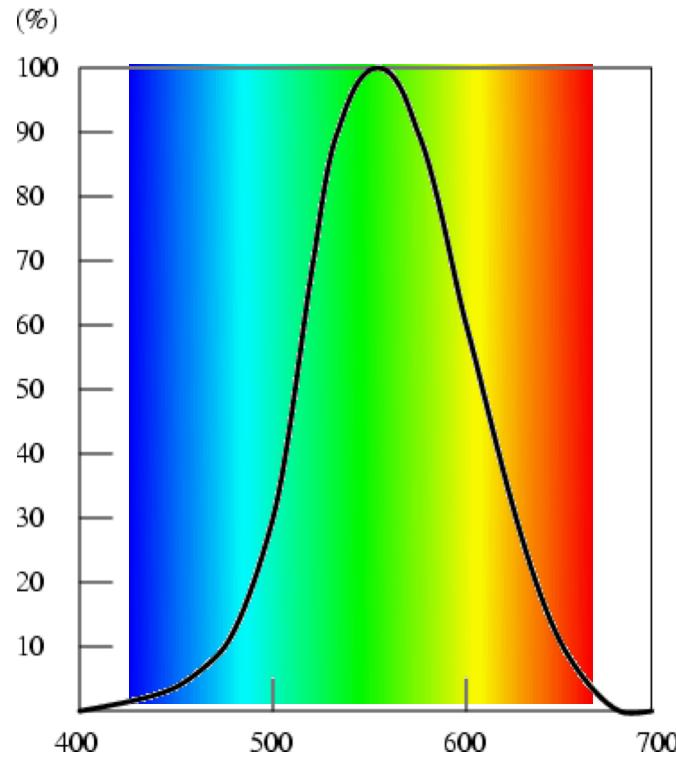
- Fourier features
- cooccurrence matrices
- filter banks (Laws, Gabor, eigenfilters)
- stochastic model



COLOUR

## The perception of brightness

- ❑ Luminous efficiency function ( $v(\lambda)$ ): relates radiometry & photometry



- ❑ C.I.E.(Commission Internationale de l'Eclairage) → standards

## Link radiometry-photometry (Watt to lumen)

*Photometry*: subjective impressions

*Radiometry*: objective, physical measurements

at 555 nm : 1lm = 1/683 W = 1.46 mW

for light with spectral composition  $c(\lambda)$  (radian flux)

$$l = k \int_{\lambda=0}^{\infty} c(\lambda) v(\lambda) d\lambda$$

with k is 683 lumens/watt

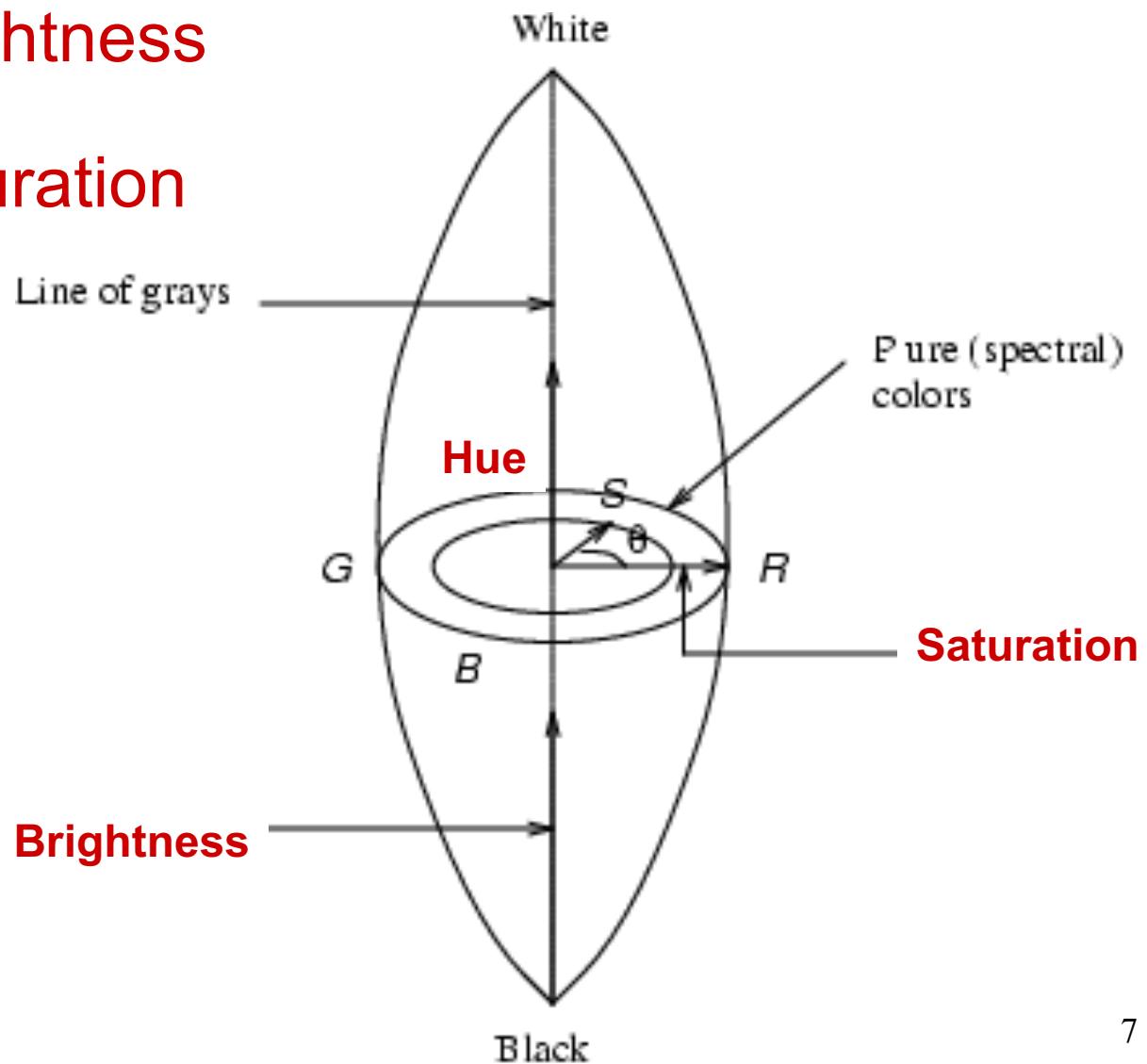
## The study of colour...

Use :

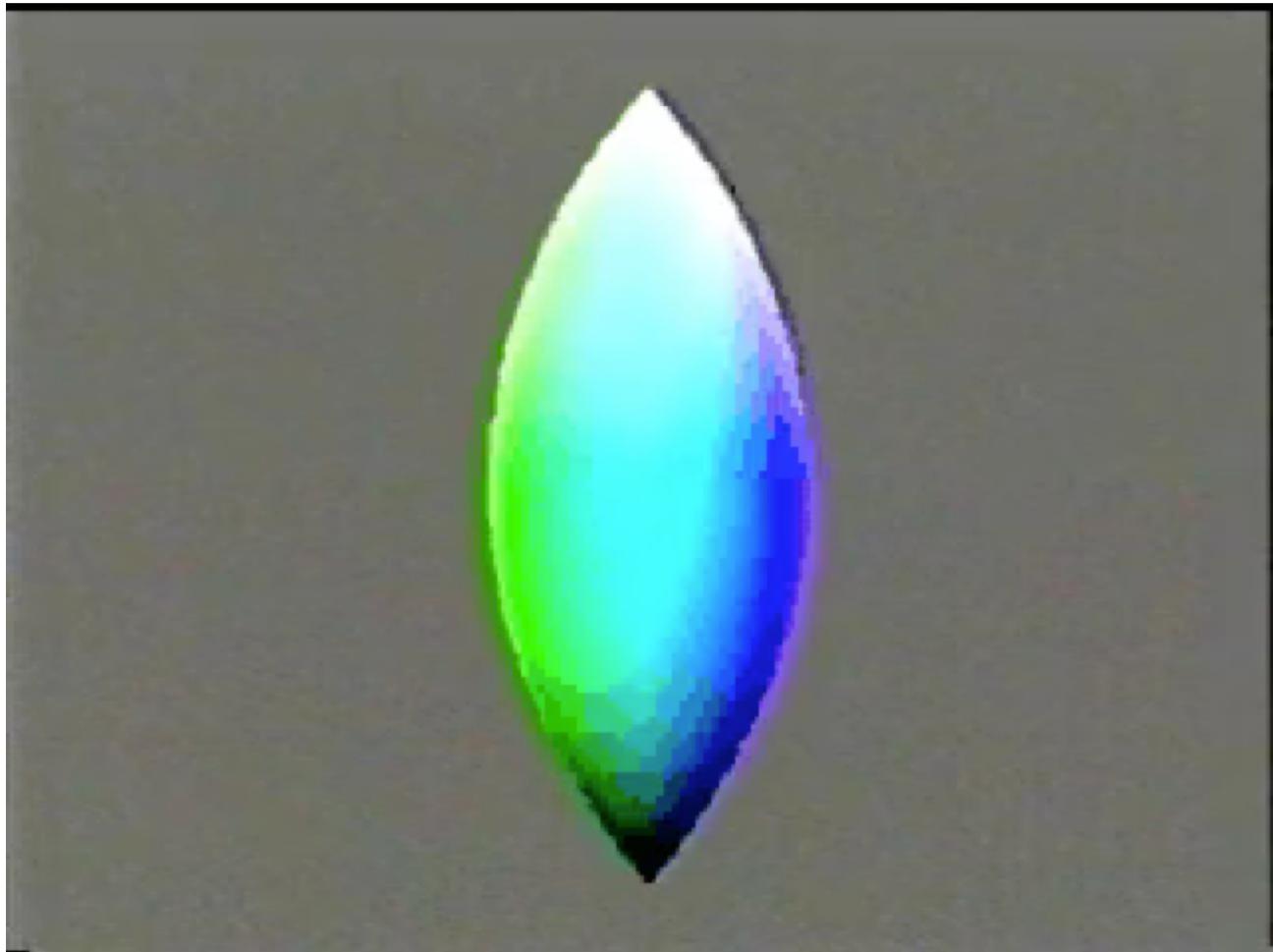
- ❑ pleasing to the eye (visualisation of results)
- ❑ characterising colours (features e.g. for recognition)
- ❑ generating colours (displays, light for inspection)
- ❑ understanding human vision

# The perceptual attributes of light (humans)

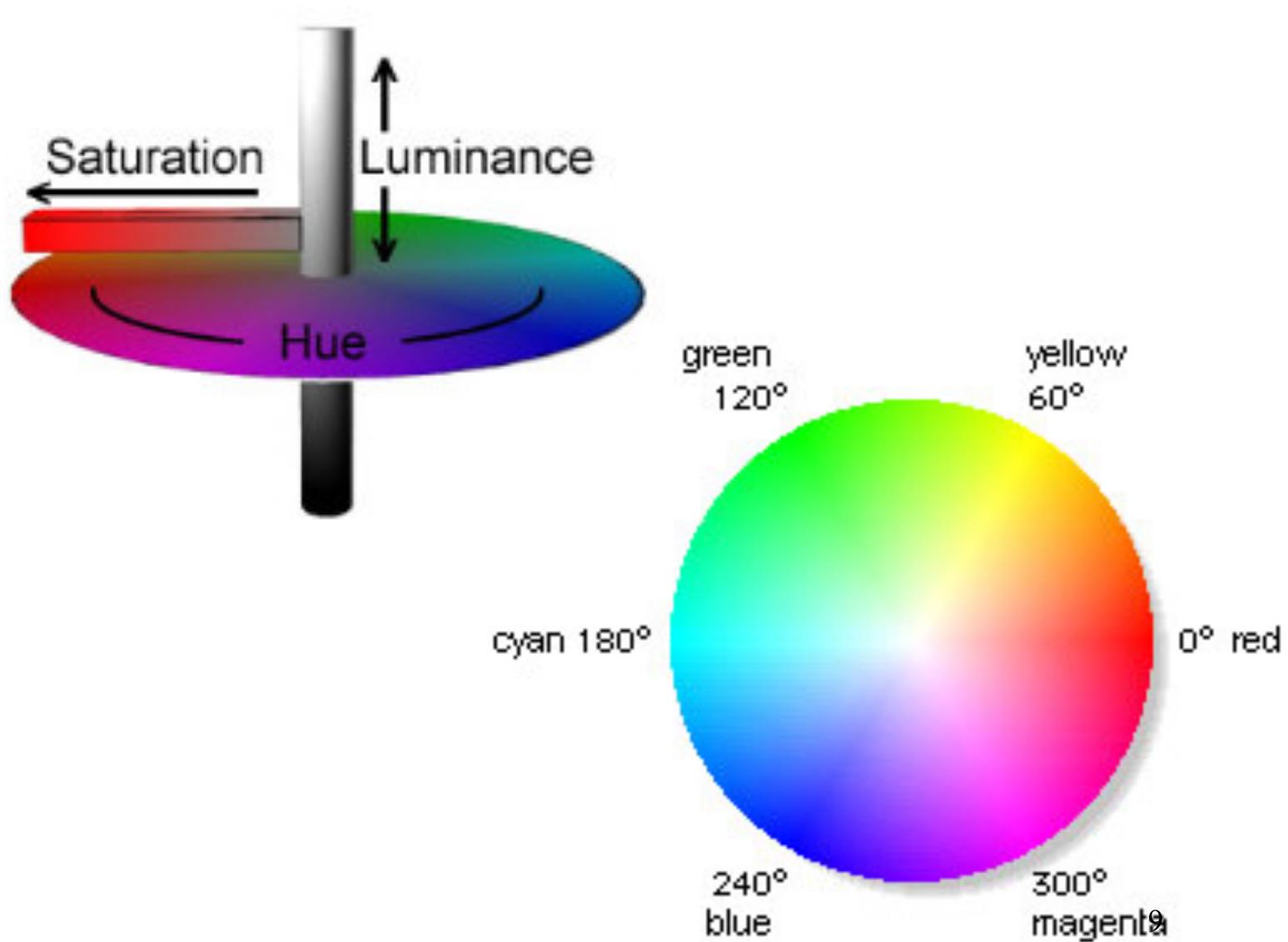
1. brightness
2. hue
3. saturation



## The C.I.E. color space



## Human perception of light = 3-dimensional



## The history of colour

Newton → spectrum



Young → tristimulus model

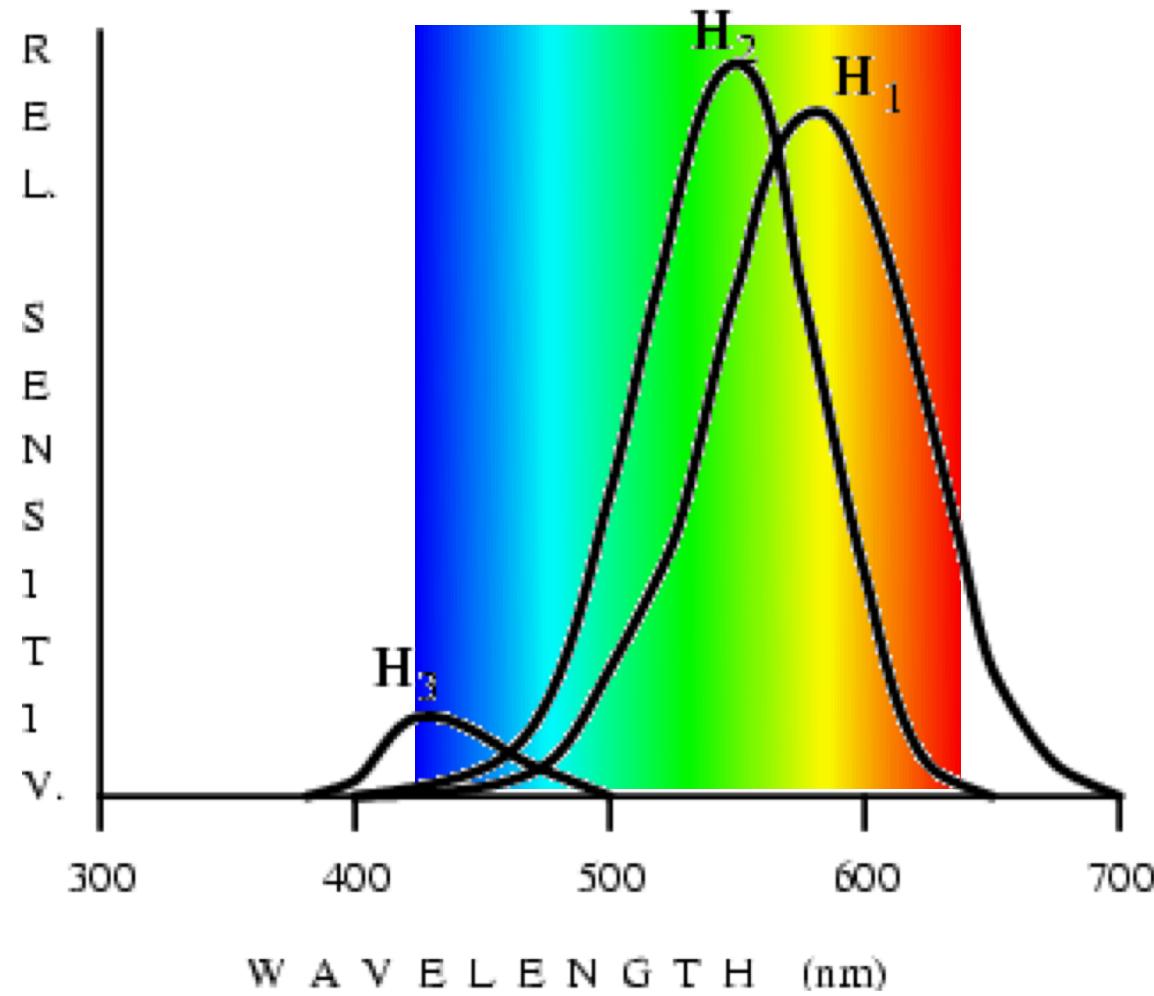


later : physiological  
underpinning :



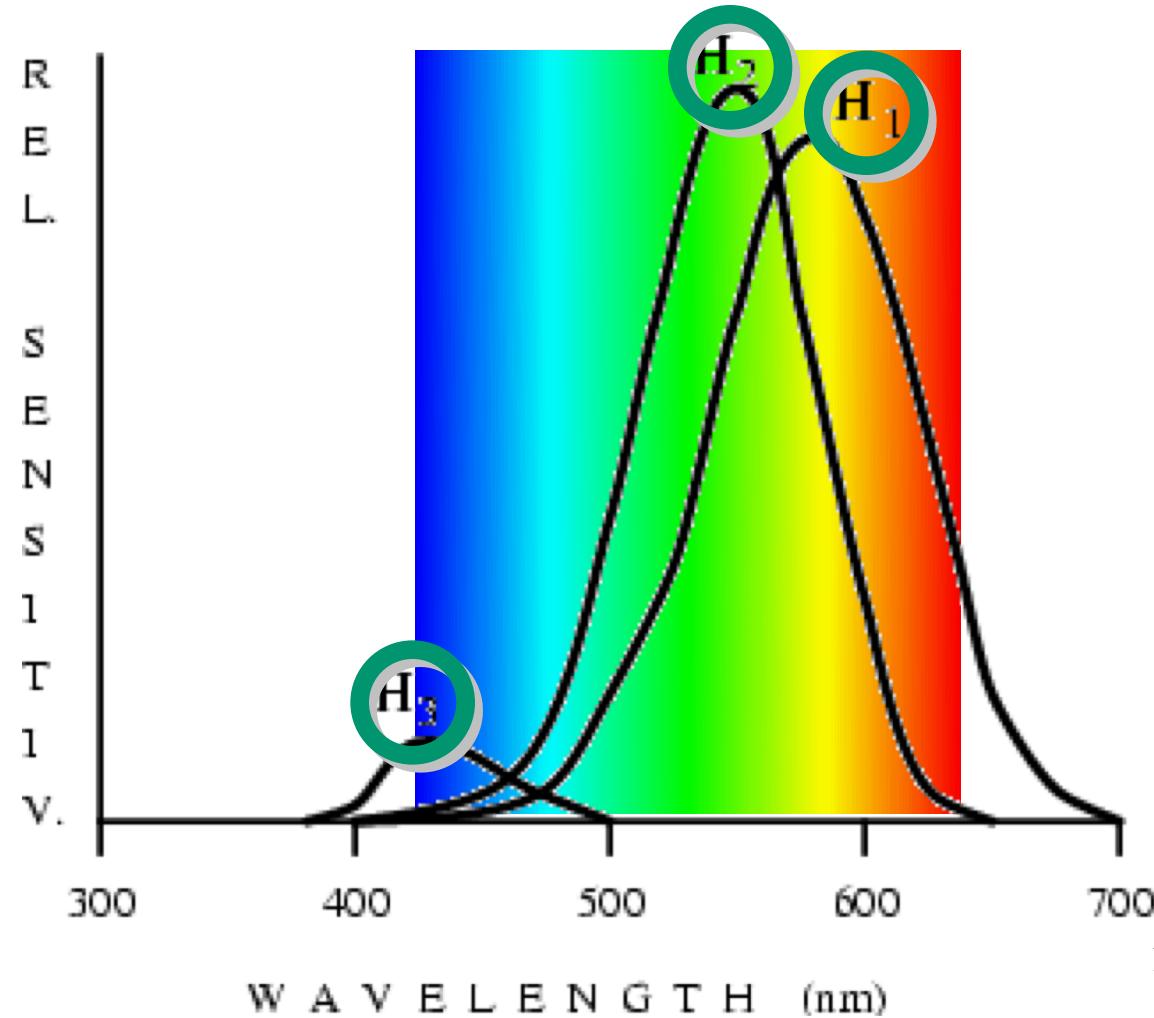
## The retinal cones

3 types : blue, green, yellow-green



## The retinal cones

3 types : blue, green, yellow-green



## Prediction of colour sensation

source with spectral radiant flux  $C(\lambda)$  produces responses  $R_i$ , with  $i = \textcolor{red}{1}, \textcolor{green}{2}, \textcolor{blue}{3}$

$$R_i(c) = \int H_i(\lambda)C(\lambda)d\lambda, i = \textcolor{red}{1}, \textcolor{green}{2}, \textcolor{blue}{3}$$

Hence, our perception of multi-spectral sources is quite impoverished:

an entire distribution over  $\lambda$  is projected onto only 3 numbers  $R$ .

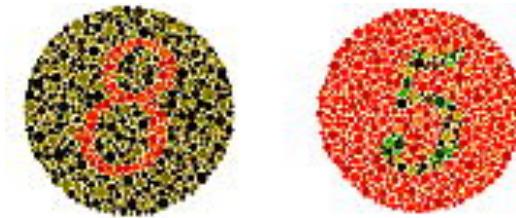


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- ❑ 2 sources with equal  $R_i$ 's  $\Rightarrow$  observed as same colour!
- ❑ luminance  $\perp$  chrominance
- ❑ 10% of population have abnormal colour vision



- ❑ several birds have 4 cone types (incl. UV)
- ❑ colour constancy

## Tristimulus representation of colour

Camera  $\Rightarrow$  tristimulus values  $\Rightarrow$  display

3 primaries  $P_j(\lambda)$ ,  $j = 1, 2, 3$

CIE primaries :  $\lambda_1 = 700$

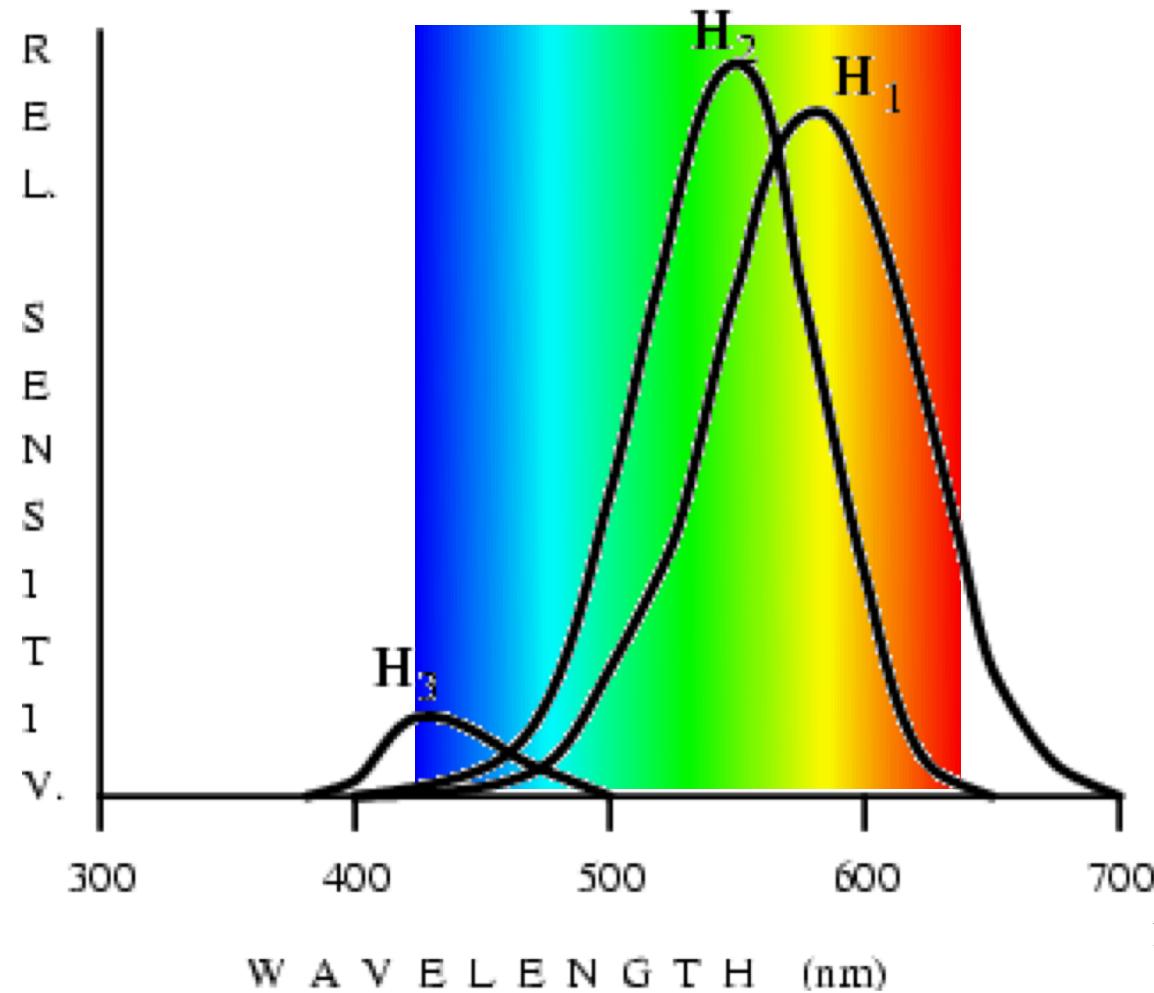
$\lambda_2 = 546.1$

$\lambda_3 = 435.8$

applications : practical primaries  
e.g. TV : EBU and NTSC

## Reminder: 3 retinal cone types

3 types : blue, green, yellow-green



## The matching of colour

source  $C(\lambda)$  matched by primaries

$$\sum_{j=1}^3 m_j P_j(\lambda)$$

$$R_i(C) = \int C(\lambda) H_i(\lambda) d\lambda$$

## The matching of colour

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## The matching of colour

source  $C(\lambda)$  matched by primaries

$$\sum_{j=1}^3 m_j P_j(\lambda)$$

$$\begin{aligned} R_i(C) &= \int \sum_{j=1}^3 m_j P_j(\lambda) H_i(\lambda) d\lambda \\ &= \sum_{j=1}^3 m_j \int H_i(\lambda) P_j(\lambda) d\lambda \end{aligned}$$



## The matching of colour

source  $C(\lambda)$  matched by primaries

$$\sum_{j=1}^3 m_j P_j(\lambda)$$

$$R_i(C) = \int \sum_{j=1}^3 m_j P_j(\lambda) H_i(\lambda) d\lambda$$

$$= \sum_{j=1}^3 m_j \underbrace{\int H_i(\lambda) P_j(\lambda) d\lambda}_{l_{i,j}}$$

can be determined “off-line”

## The math

extremely simple : linear equations

$$\sum_{j=1}^3 m_j l_{i,j} = R_i$$

implies inverting the matrix :  
independent primaries!

also linear transform between the  $m_j$ 's  
for different choices of primaries:

$$L m = R$$

$$L' m' = R$$

gives

$$m' = L'^{(-1)} L m$$

## Tristimulus values

“white” considered a reference :  
specify relative values w.r.t.  $m_j$ s for white :  $w_j$

$$\textcolor{red}{Tristimulus values : } T_j = \frac{m_j}{w_j}$$

The scaling preserves the linearity

CIE tristimulus values : **R**, **G**, **B**  
(for CIE white = flat spectrum &  $w_1 = w_2 = w_3$ )

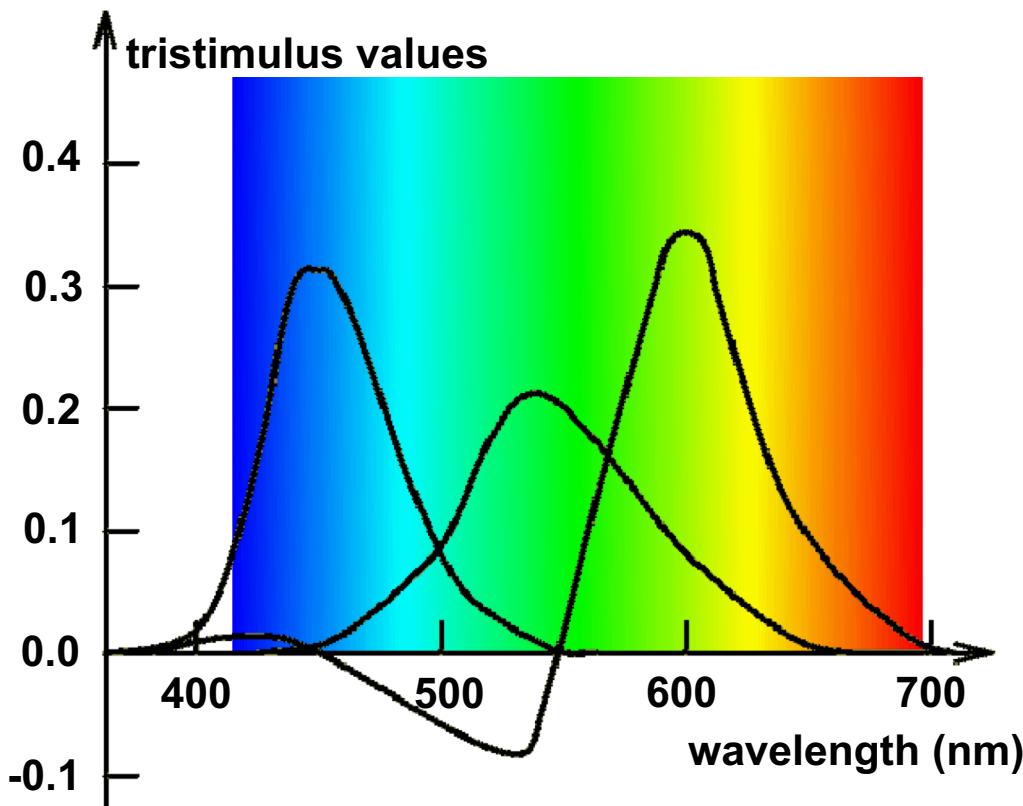
Note that for white  $T_1 = T_2 = T_3 = 1$

## Spectral matching curves

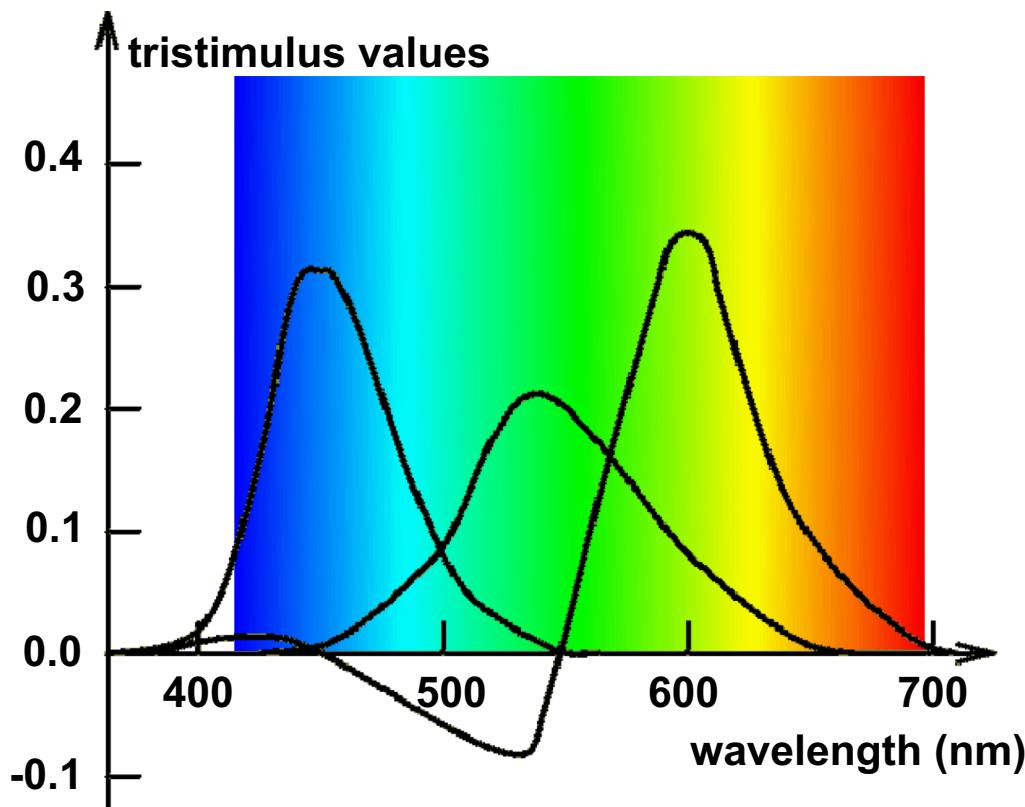
*Spectral matching curves*  $T_j(\lambda)$ :  
values for monochromatic sources

$$R_i(C_\lambda) = H_i(\lambda) = \sum_{j=1}^3 m_j l_{i,j} = \sum_{j=1}^3 w_j l_{i,j} T_j(\lambda)$$

for the CIE primaries:



## Interpretation of these curves



negative values : colours that cannot be produced

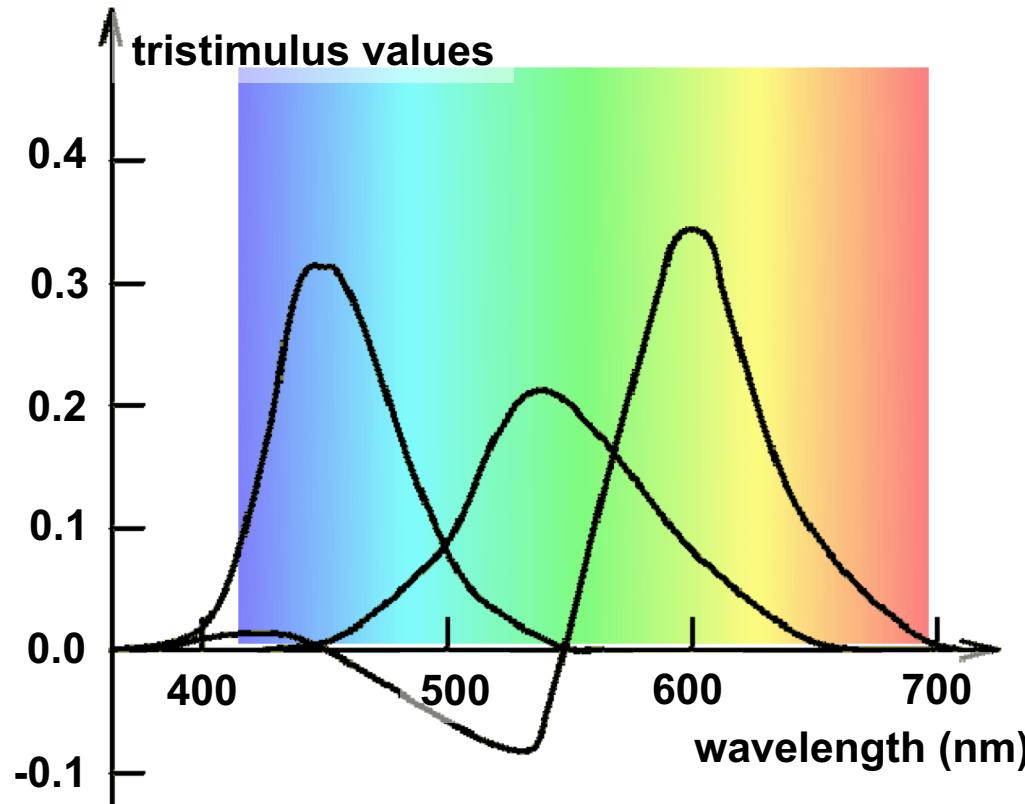
In such case:

$\text{mix}(\text{target}, \text{neg. primary}) = \text{mix}(\text{pos. primaries})$

for any primary triple some colours cannot be produced

## Interpretation of these curves for arbitrary source $C(\lambda)$ :

$$T_j(C) = \int C(\lambda)T_j(\lambda)d\lambda$$



## Chromaticity coordinates

tristimulus values still contain brightness info

chrominance info pure : normalising the tristimulus values



*chromaticity coordinates :*

$$t_j = \frac{T_j}{T_1 + T_2 + T_3}$$

$t_1 + t_2 + t_3 = 1$  allows to eliminate one

2 chromaticity coordinates specify saturation and hue

## Chromaticity coordinates

tristimulus values still contain brightness info

chrominance info pure : normalising the tristimulus values



*chromaticity coordinates :*

$$t_j = \frac{T_j}{T_1 + T_2 + T_3}$$

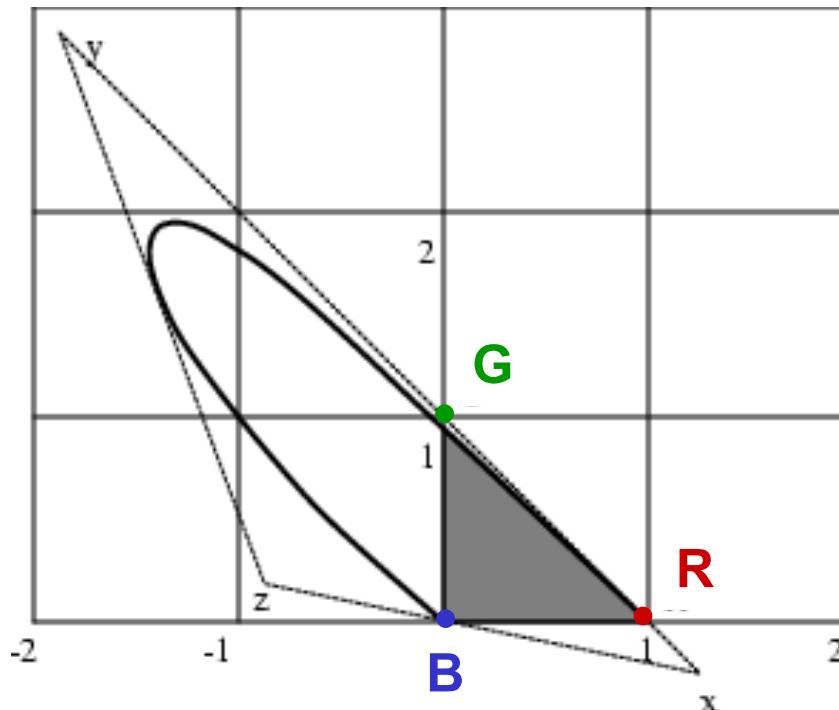
Note that for white  $t_1 = t_2 = t_3 = 1/3$

## CIE chromaticity diagram

chromaticity coordinates ( $r, g$ ) for CIE primaries :

$$r = \frac{R}{R + G + B} \quad g = \frac{G}{R + G + B}$$

The corresponding colour space :



## CIE x-y coordinates

In order to get rid of the negative values :  
virtual tristimulus colour system X,Y,Z  
*(no such physically realizable primaries exist !)*

linear transf. from R,G,B to X,Y,Z coordinates :

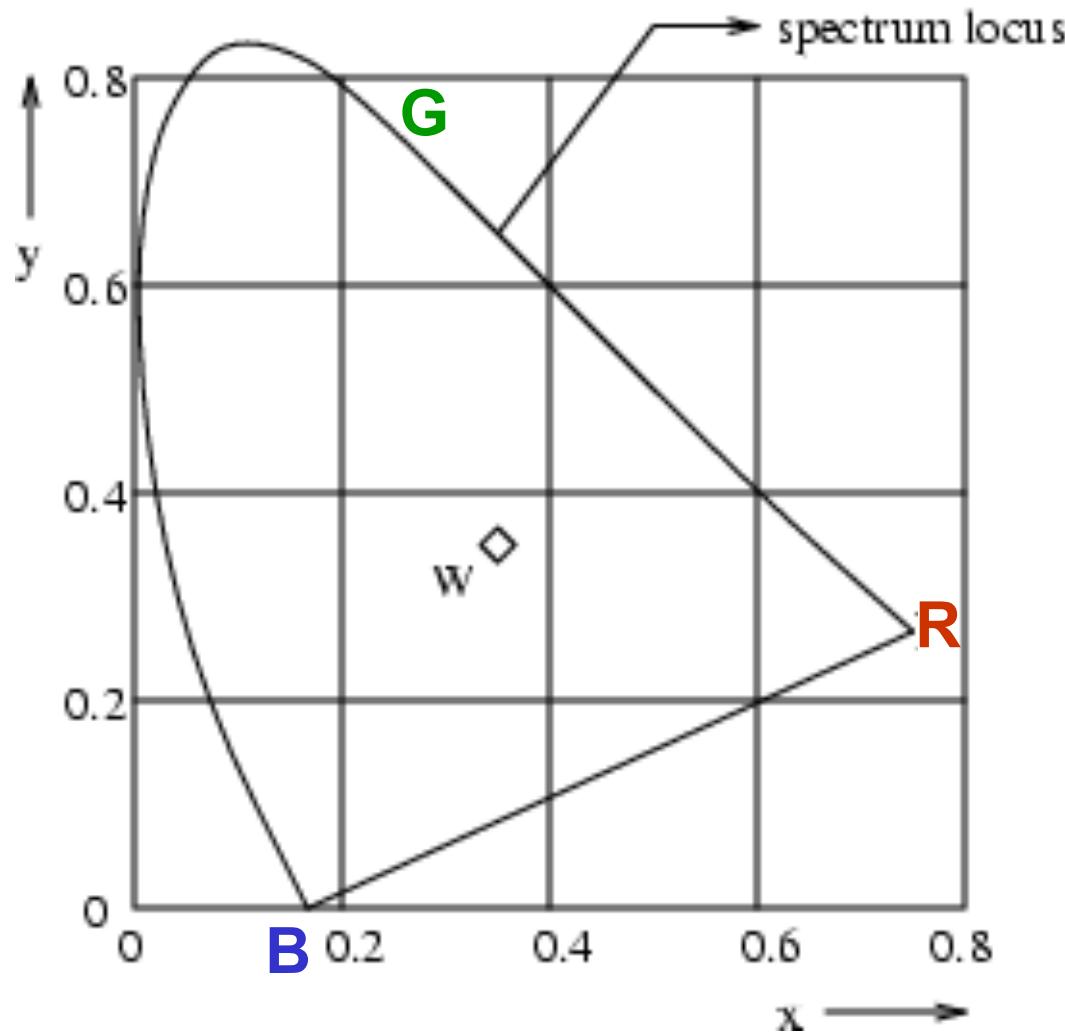
$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0.490 & 0.310 & 0.200 \\ 0.177 & 0.813 & 0.011 \\ 0.000 & 0.010 & 0.990 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

chosen as to make Y represent luminance

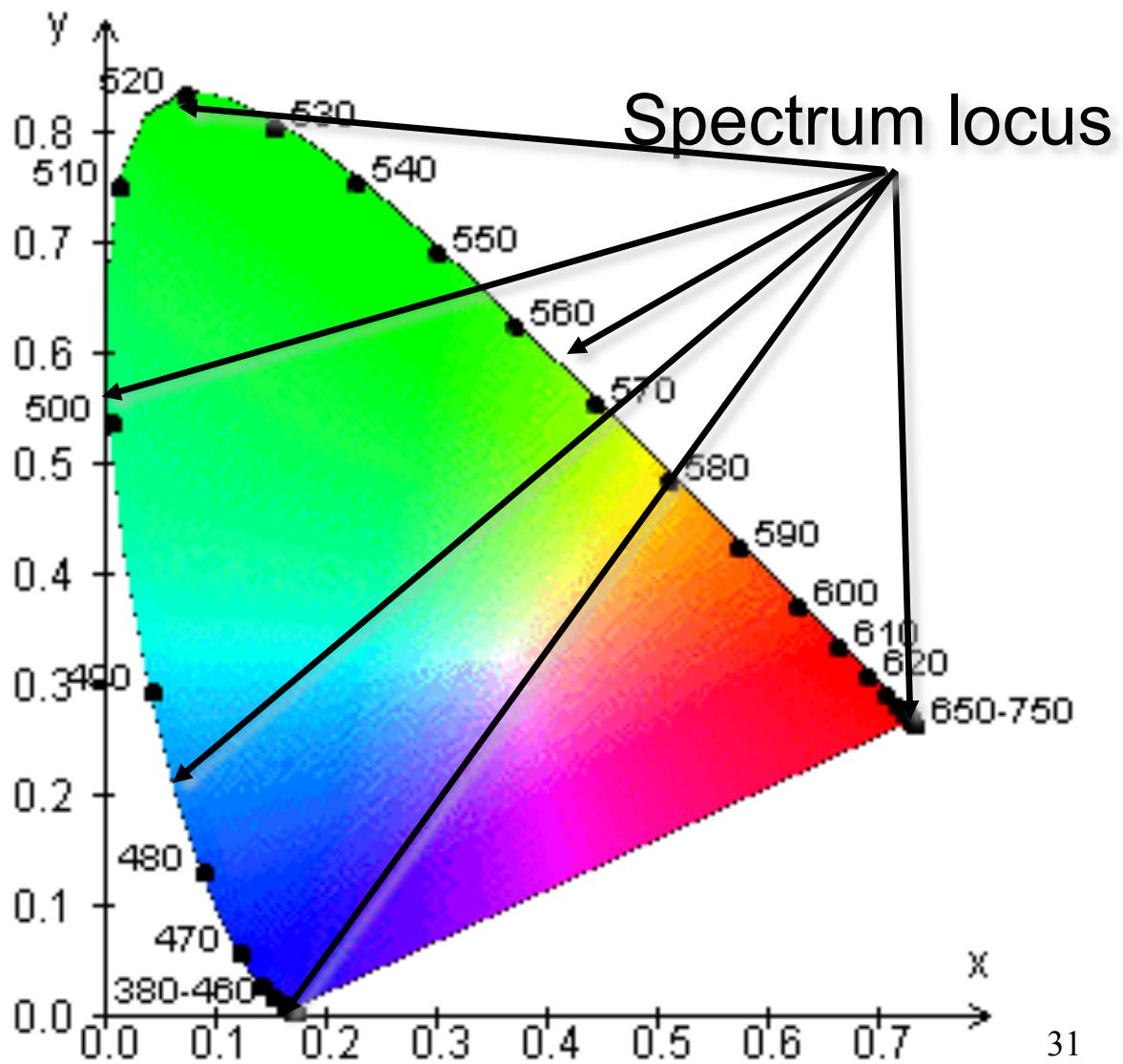
white (R=G=B=1) mapped to X=Y=Z=1

$$x = \frac{X}{X + Y + Z} \qquad y = \frac{Y}{X + Y + Z}$$

## CIE x,y colour triangle



## CIE x,y colour triangle



## TV primaries

the EBU primaries have coordinates

$$R_r : \quad x = 0.64 \quad y = 0.33$$

$$G_r : \quad x = 0.29 \quad y = 0.60$$

$$B_r : \quad x = 0.15 \quad y = 0.06$$

the NTSC primaries have coordinates

$$R_N : \quad x = 0.67 \quad y = 0.33$$

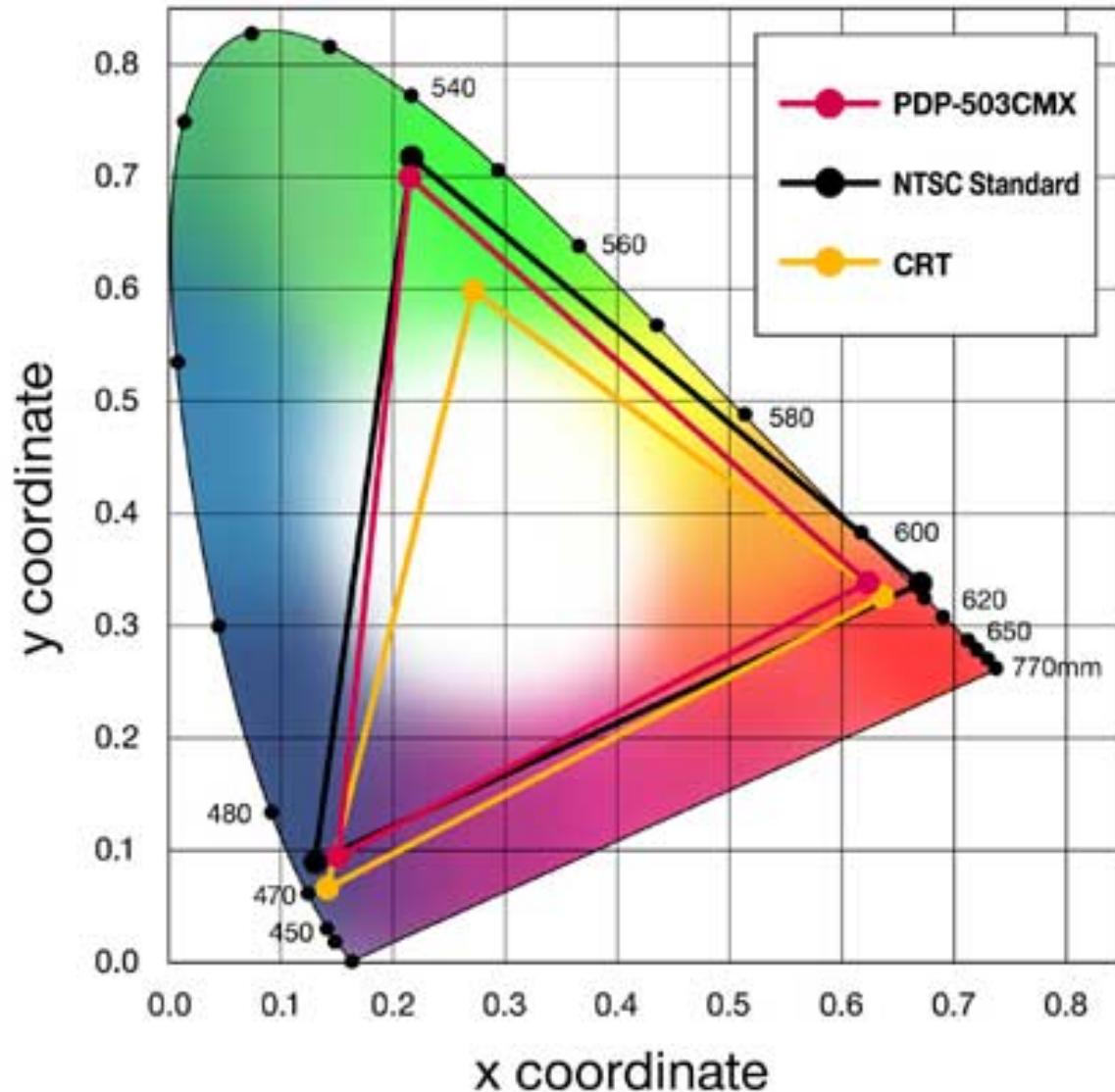
$$G_N : \quad x = 0.21 \quad y = 0.71$$

$$B_N : \quad x = 0.14 \quad y = 0.08$$



## primaries

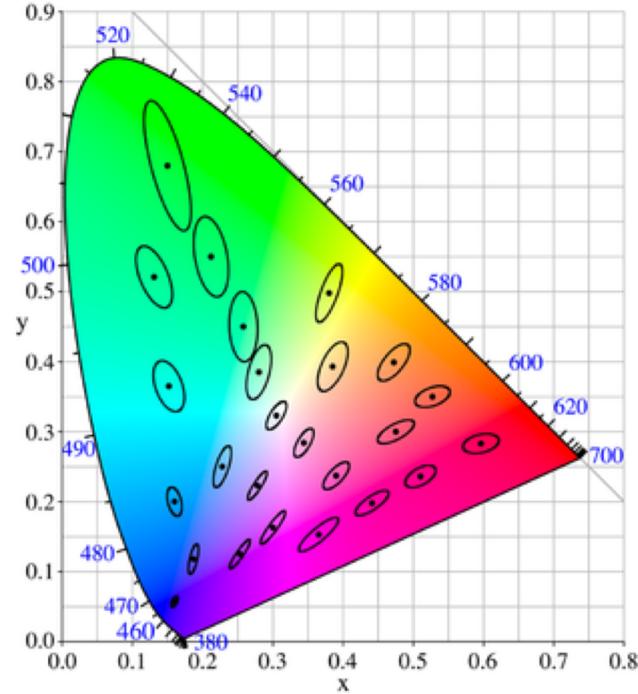
### CIE Chromaticity Coordinates



## Notes

Minimize colours outside the triangle!

Area dubious criterion :  
projective transf. between chromaticity coordinates  
distance in triangle no faithful indication of  
perceptual difference



pure spectrum colours (on the spectrum locus) are rare  
in nature



## Chromaticity coordinate transitions

So, colour coordinates need for their definition  
3 primaries + white : 4 points

4 points define a projective frame :

primaries  $\Rightarrow (0,0), (1,0), (0,1)$

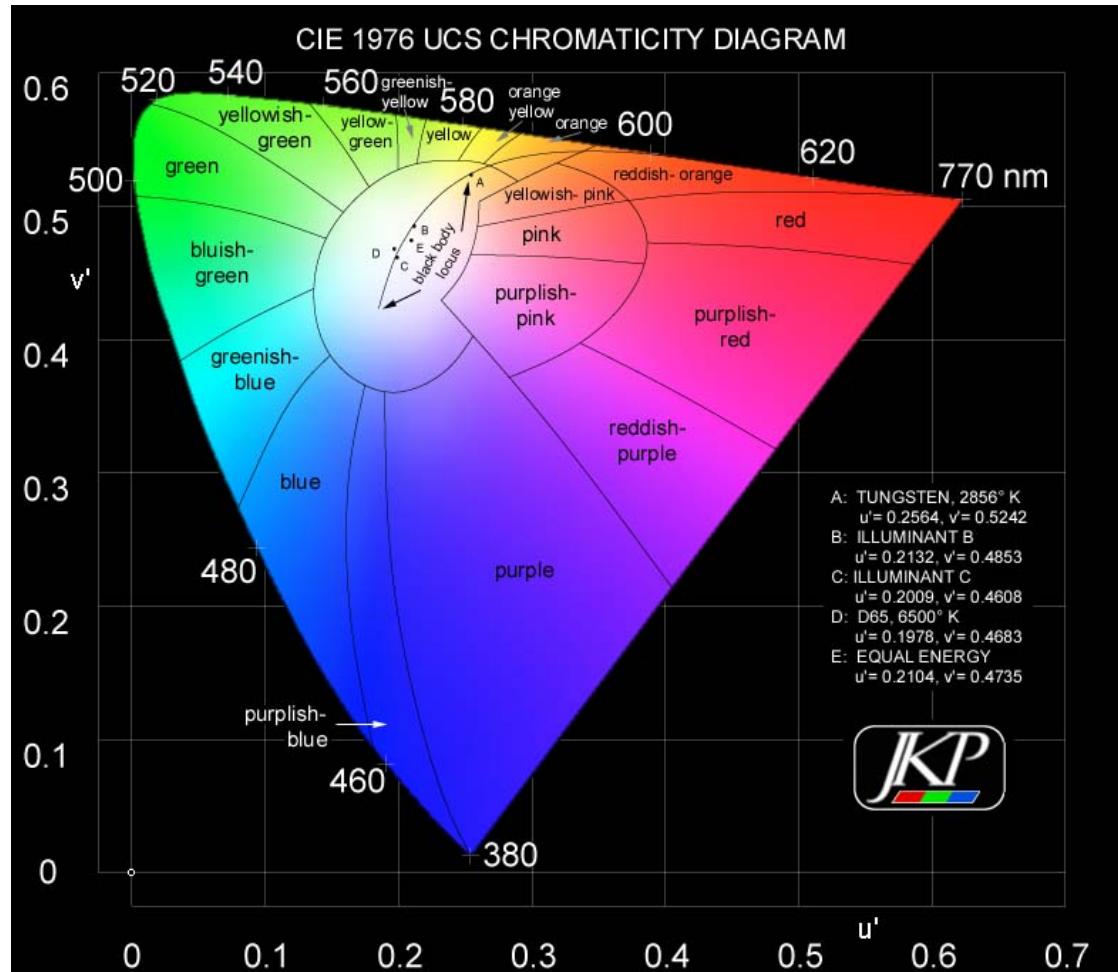
white  $\Rightarrow (0.33, 0.33)$

a chromaticity coordinate transformation can be shown to be projective, i.e. non-linear

# CIE u-v color coordinates

$u - v$  diagram +/- faithfully represents perceptual distance :

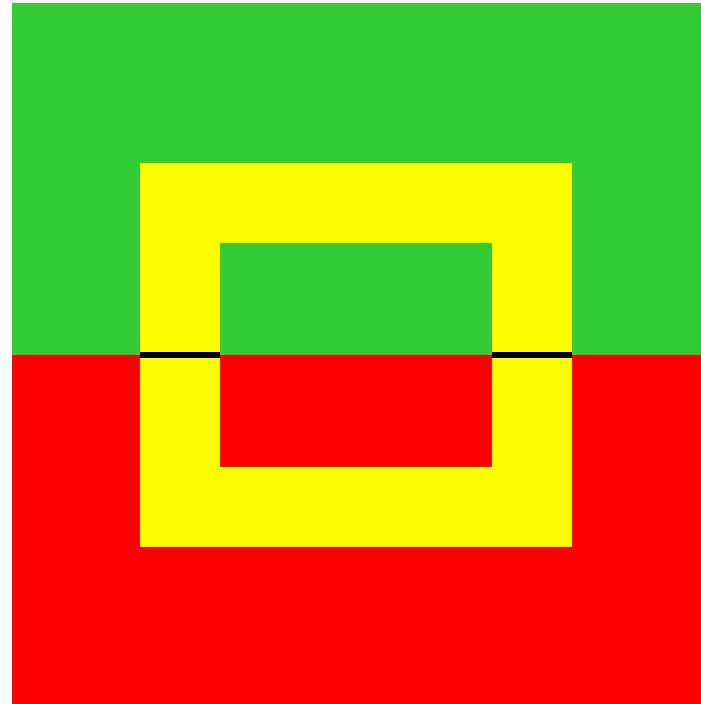
$$u = \frac{4x}{-2x + 12y + 3} \quad v = \frac{6y}{-2x + 12y + 3}$$



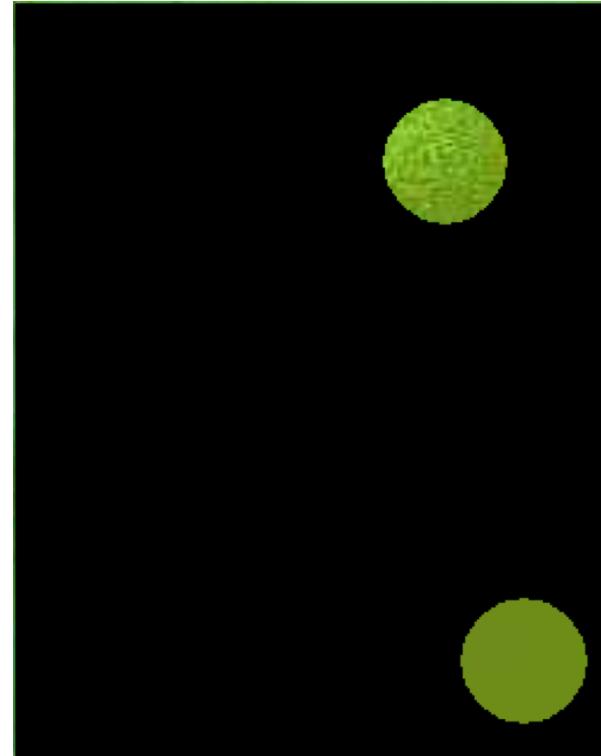
## Using colour as a feature:

- colour constancy
- illumination invariant colour features

## Koffka ring with colours



## Colour constancy



## Colour constancy

Patches keep their colour appearance even if they reflect differently (e.g. the hat)

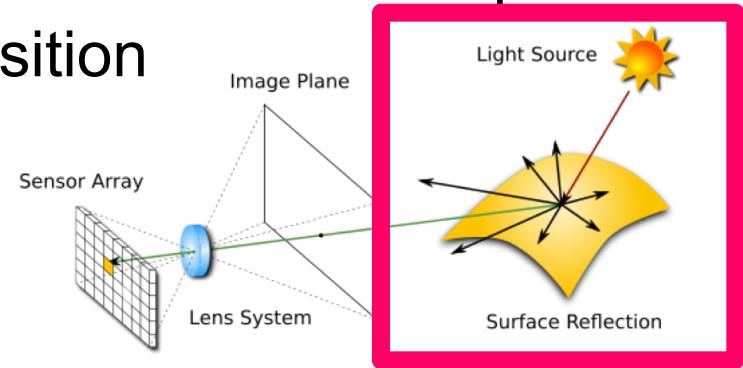
Patches change their colour appearance if they reflect identically but surrounding patches reflect differently (e.g. the background)

There is more to colour perception than 3 cone responses

Edwin Land performed in-depth experiments (psychophysics)

## Colour constancy - notes

The colour of a surface is the result of the product of spectral reflectance and spectral light source composition



Our visual system can from a single product determine the two factors, it seems

The colour of the light source can be guessed via that of specular reflections, but the visual system does not critically depend on this trick

## On the menu:

- colour constancy
- illumination invariant colour features

## Illumination invariant colour features

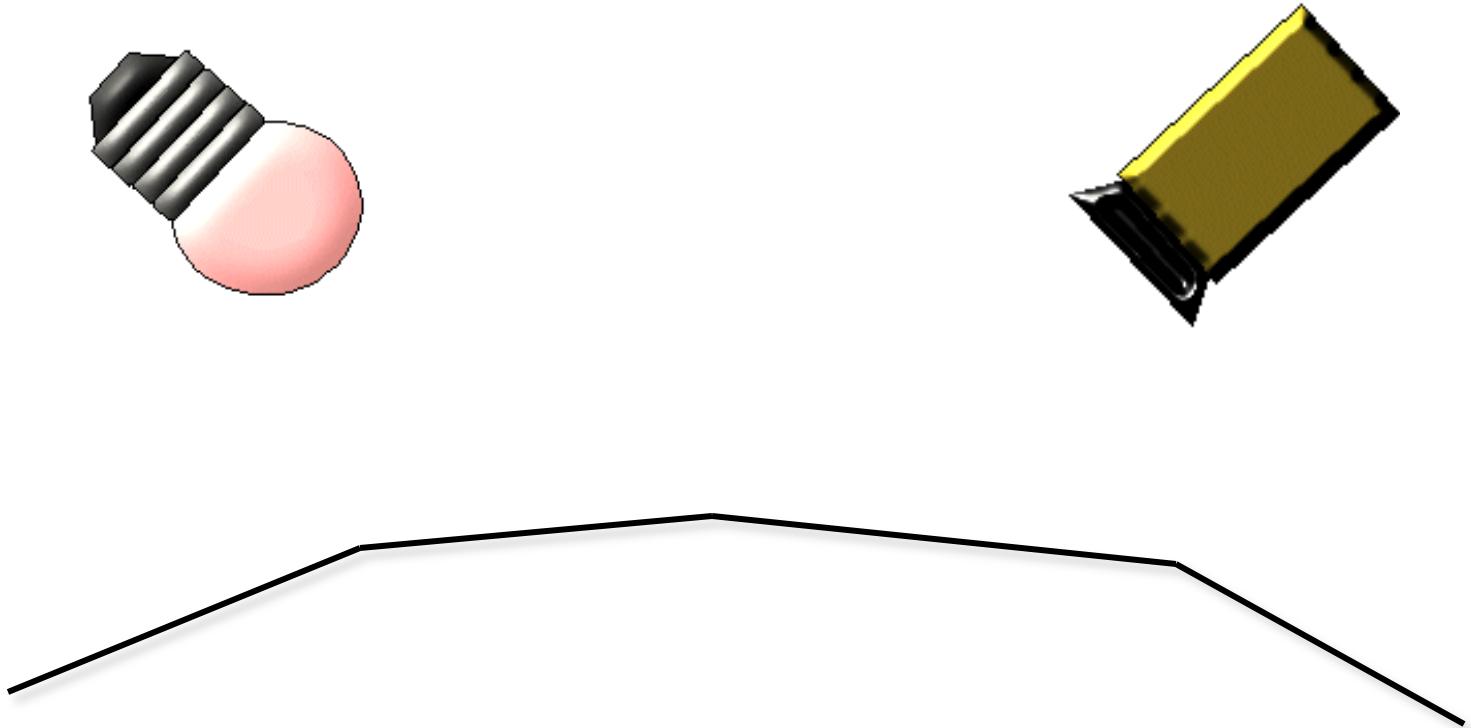
Extracting the true surface colour under varying illumination - as the HVS can - is very difficult

A less ambitious goal is to extract colour features that do not change with illumination

- 1) Spectral or 'internal' changes
- 2) Geometric or 'external' changes
- 3) Spectral + geometric changes

## Illumination invariant colour features

### 1) Spectral changes



# Illumination invariant colour features

## 1) Spectral changes

Let  $I_R, I_G, I_B$  represent the irradiances at the camera for red, green, blue

A simple model: the irradiances change by  $\alpha, \beta, \gamma$ :  $(I'_R, I'_G, I'_B) = (\alpha I_R, \beta I_G, \gamma I_B)$

Consider irradiances at 2 points:  
 $I_{R1}, I_{G1}, I_{B1}$  and  $I_{R2}, I_{G2}, I_{B2}$

$$I'_{R1} / I'_{R2} = (\alpha I_{R1}) / (\alpha I_{R2}) = I_{R1} / I_{R2}$$

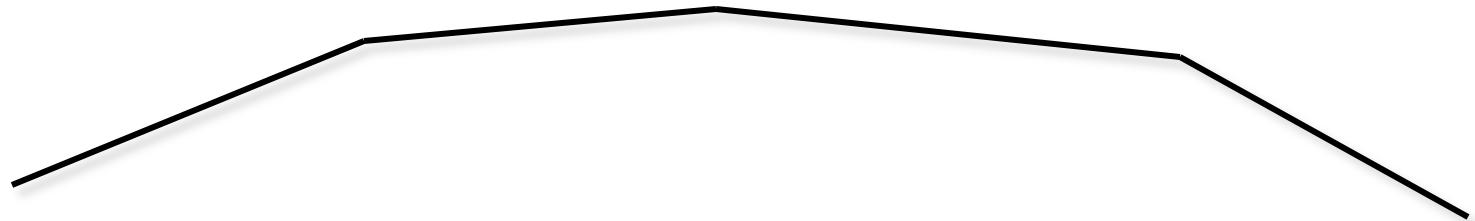
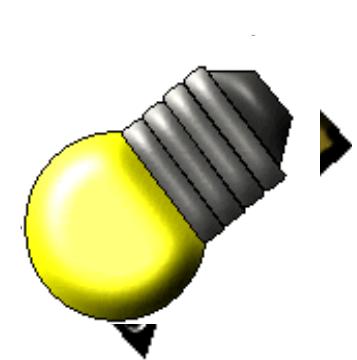
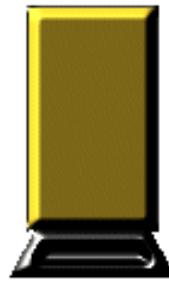
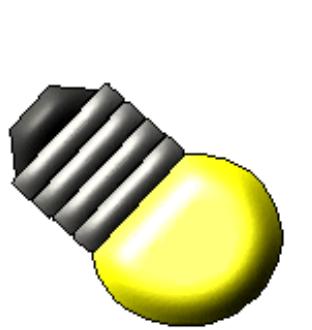
# Illumination invariant colour features

For a camera with a non-linear response:

$$(\alpha I_{R1})^\gamma / (\alpha I_{R2})^\gamma = (I_{R1})^\gamma / (I_{R2})^\gamma$$

## Illumination invariant colour features

### 2) Geometric changes



# Illumination invariant colour features

## 1) Geometric changes

$$(I'_R, I'_G, I'_B) = (s(x, y)I_R, s(x, y)I_G, s(x, y)I_B)$$

$$I'_R / I'_G = I_R / I_G$$

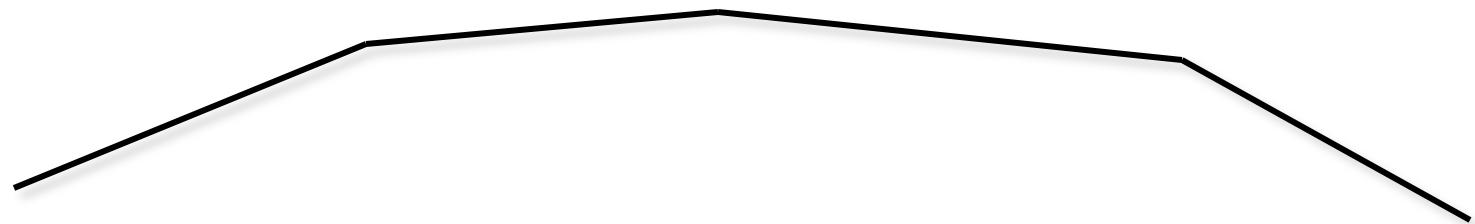
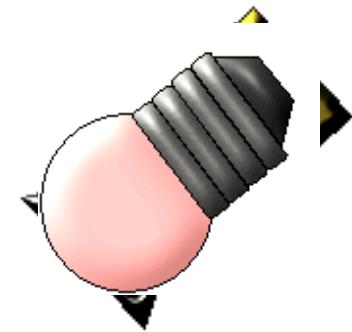
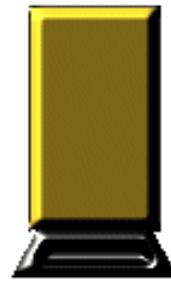
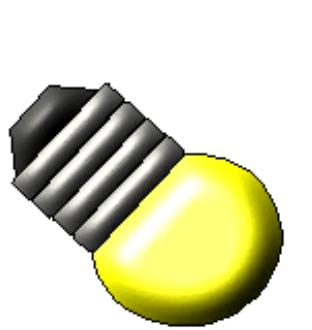
and

$$I'_R / I'_B = I_R / I_B$$

are invariant

## Illumination invariant colour features

### 3) Spectral + geometric changes



## Illumination invariant colour features

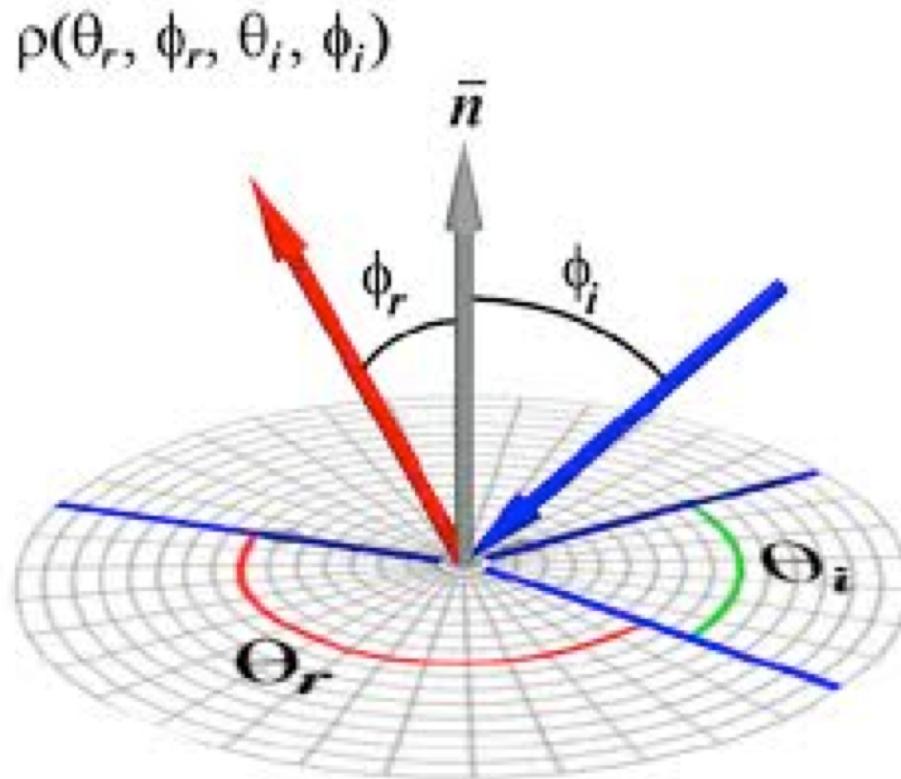
### 3) Geometric + spectral changes

$$\frac{I'_{R1} I'_{G2}}{I'_{R2} I'_{G1}} = \frac{\alpha s(x_1, y_1) I_{R1} \beta s(x_2, y_2) I_{G2}}{\alpha s(x_2, y_2) I_{R2} \beta s(x_1, y_1) I_{G1}} = \frac{I_{R1} I_{G2}}{I_{R2} I_{G1}}$$

for points on both sides of a colour edge  
 $s(x_1, y_2) \cong s(x_2, y_2)$  and hence  $I_{R1} / I_{R2}$   
is invariant

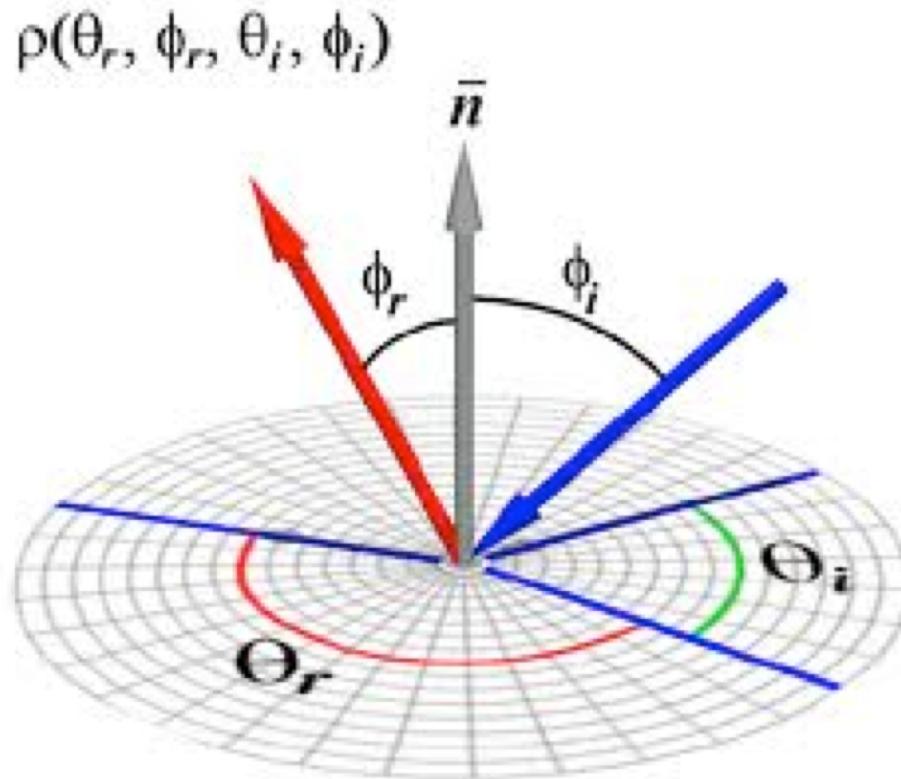
# The elusive BRDF

Bidirectional Reflection Distribution Function  
.... for different wavelengths



## The elusive BRDF

A 4D function, specifying the radiance for an outgoing direction given an irradiance for an incoming direction, relative to the normal and ideally for 1 wavelength at a time



## Mini-dome to study reflectance



KATHOLIEKE UNIVERSITEIT  
**LEUVEN**

The logo of KU Leuven, featuring the university's name in white text on a black background. The word "KATHOLIEKE" is in a smaller font above the word "LEUVEN", which is in a large, bold, sans-serif font.

# Mini-dome to study reflectance



# Mini-dome to study reflectance



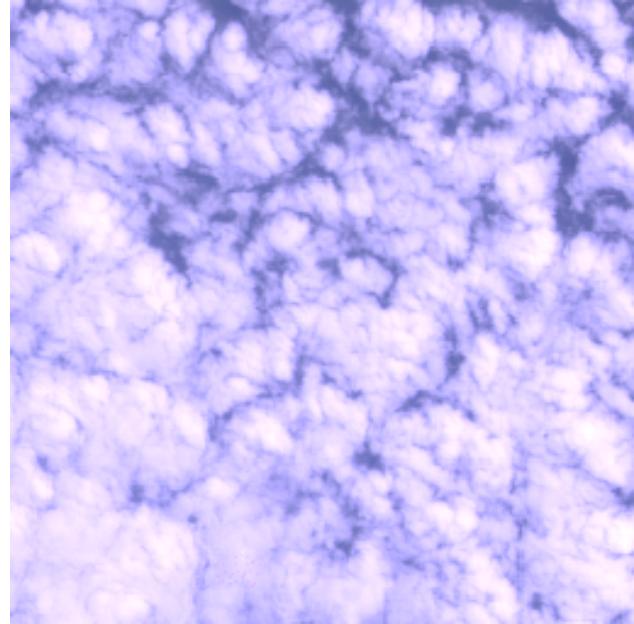
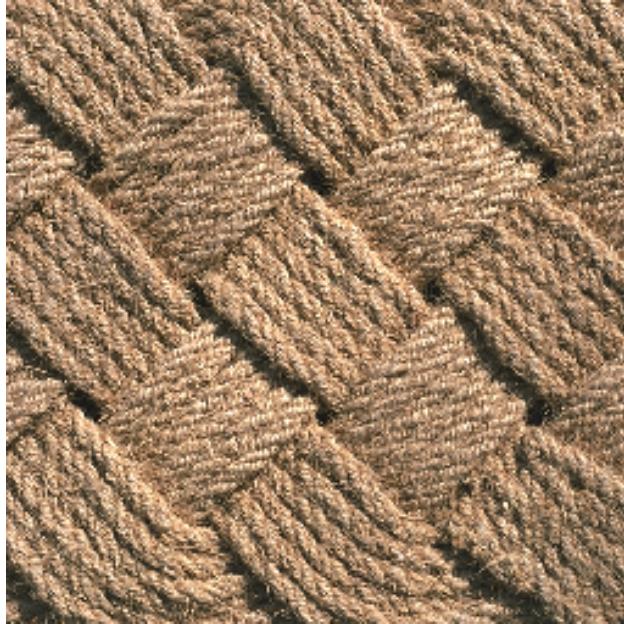
REAL

SIMULATED<sup>55</sup>



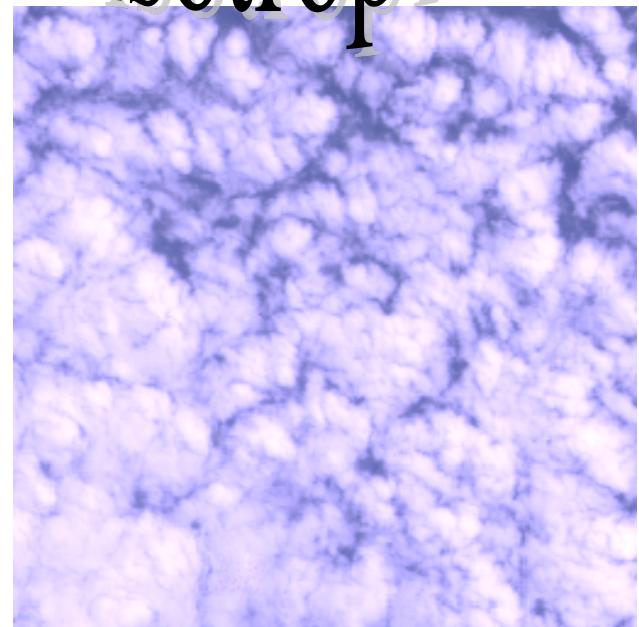
TEXTURE

# Example textures



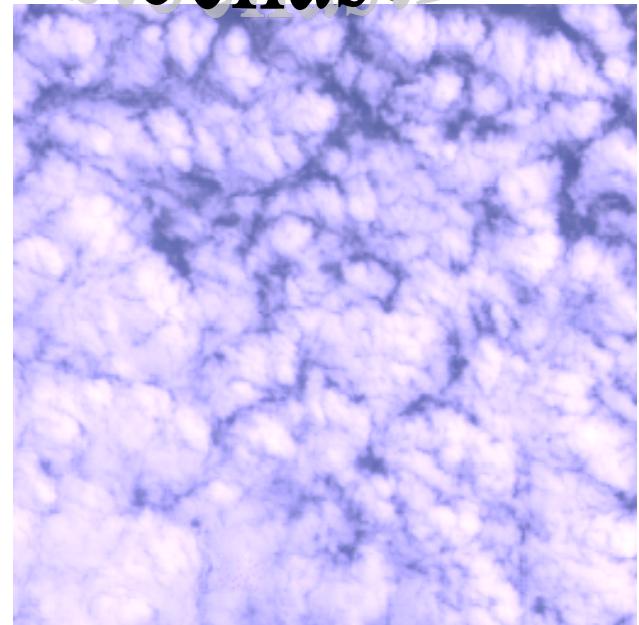
# Texture characteristics

oriented vs. isotropic



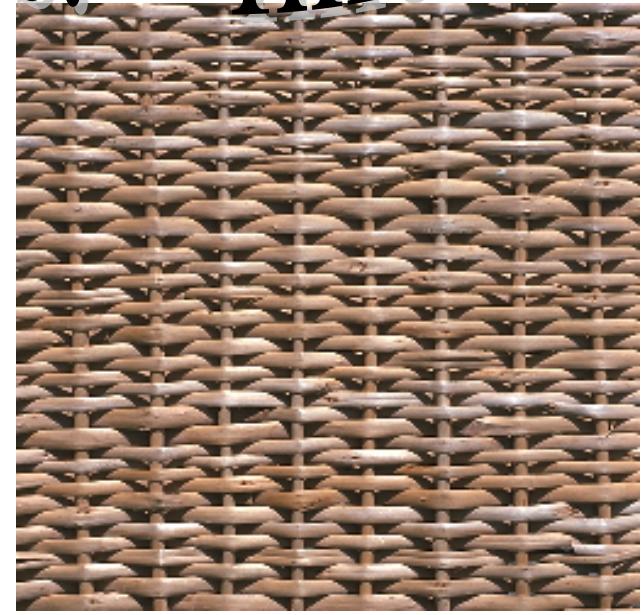
# Texture characteristics

regular    vs.    stochastic



# Texture characteristics

coarse      vs.      fine



## On the menu:

- Fourier features
- cooccurrence matrices
- filter banks (Laws, Gabor, eigenfilters)
- stochastic models)

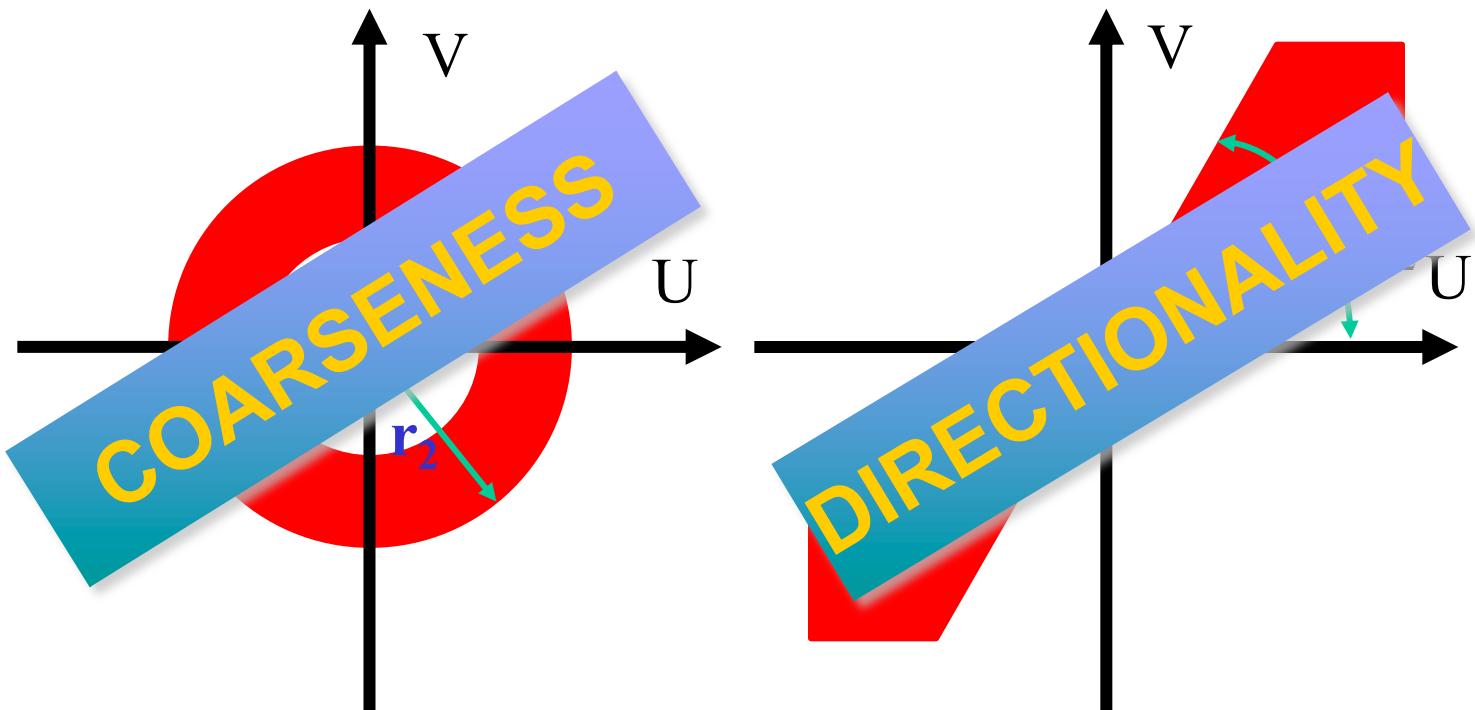
## Fourier features

Based on the integration of regions of the Fourier power spectrum  $\int_A \int |F(u, v)|^2 du dv$

Intuitively appealing

- peaks if periodic
- mostly low/high freq. if coarse resp. fine
- the sine patterns each have an orientation

# Fourier features



$$r_1^2 \leq u^2 + v^2 < r_2^2$$

$$\theta_1 \leq \arctan\left(\frac{u}{v}\right) < \theta_2$$

## Fourier features

THE FOURIER TRANSFORM COLLECTS INFORMATION GLOBALLY OVER THE ENTIRE IMAGE

NOT GOOD FOR SEGMENTATION OR INSPECTION

## On the menu:

- Fourier features
- cooccurrence matrices
- filter banks (Laws, Gabor, eigenfilters)
- stochastic models

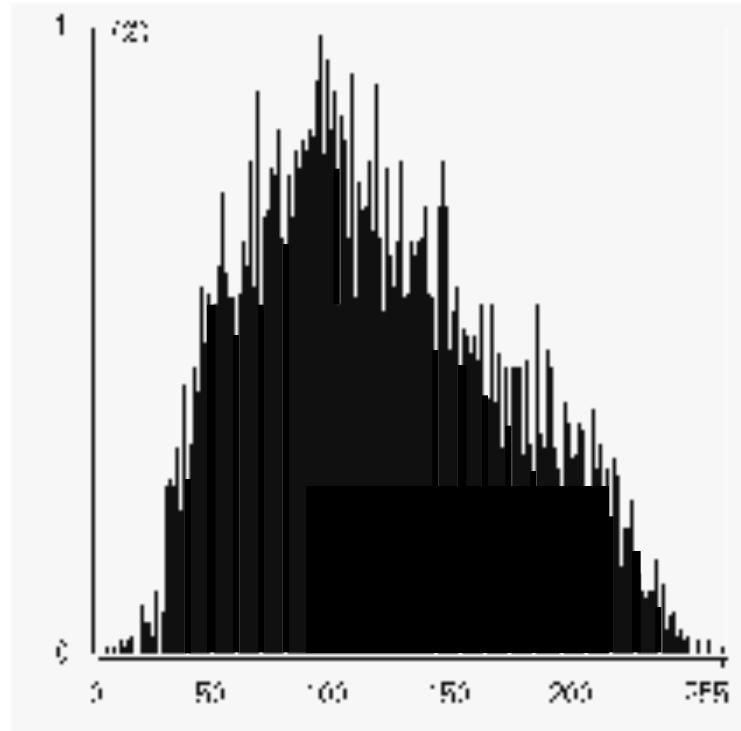
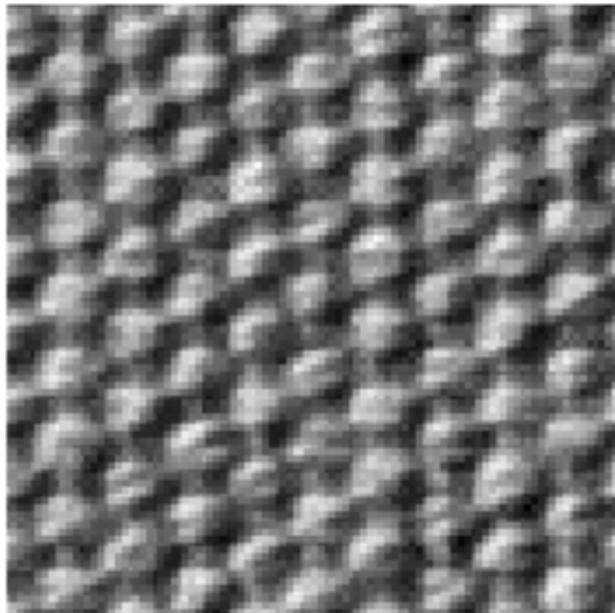
## Histograms: principle

Intensity probability distribution

Captures global brightness information in a compact, but incomplete way

Doesn't capture spatial relationships

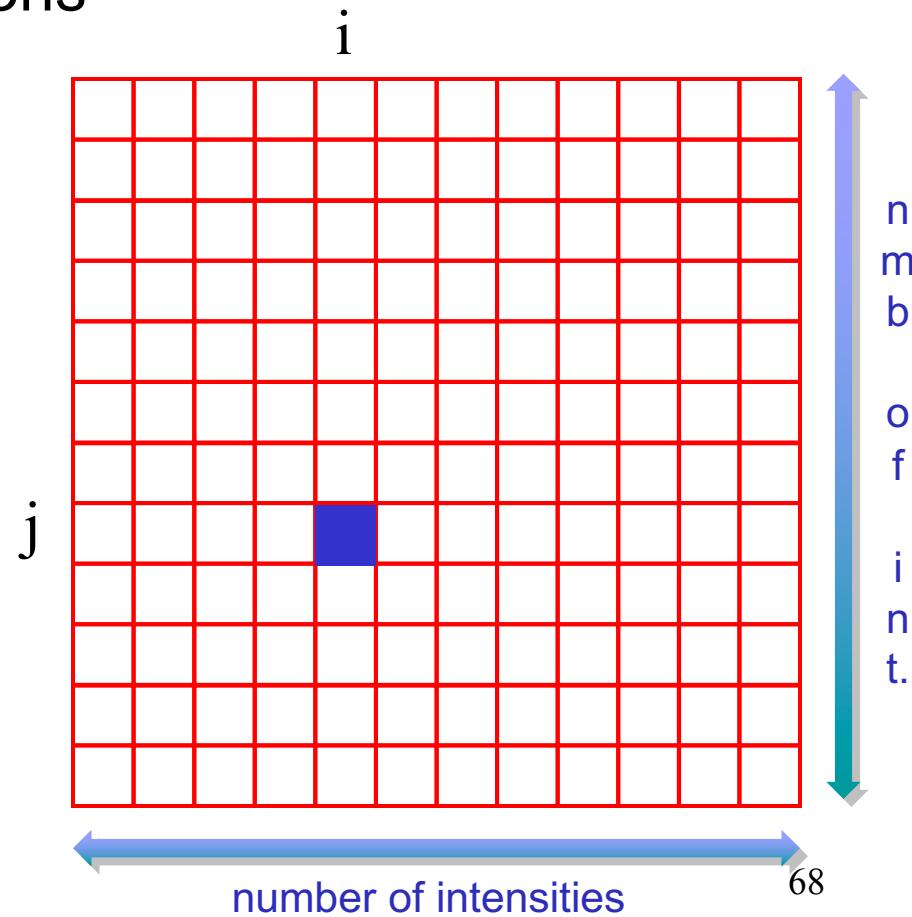
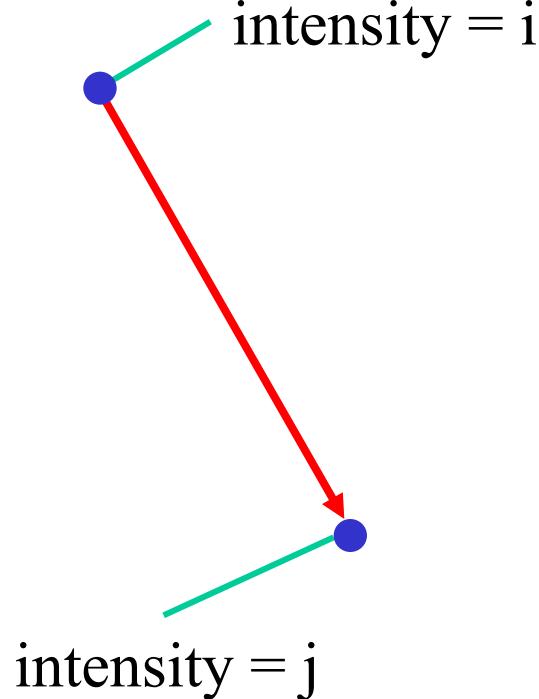
## Histograms : example



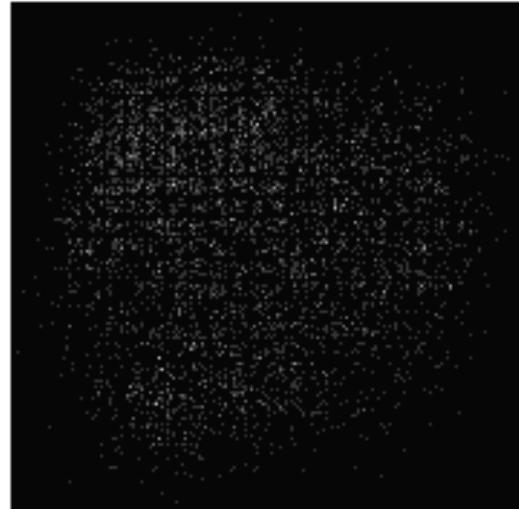
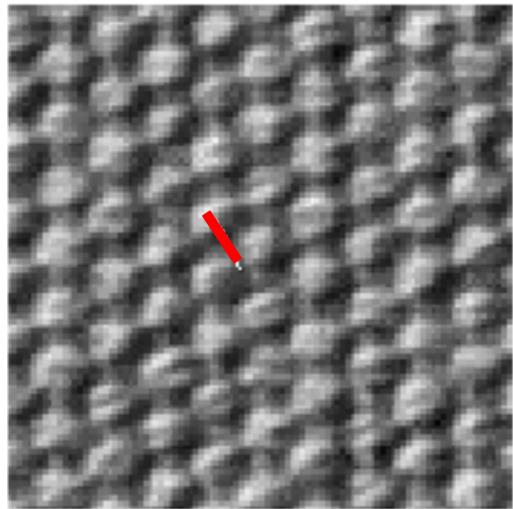
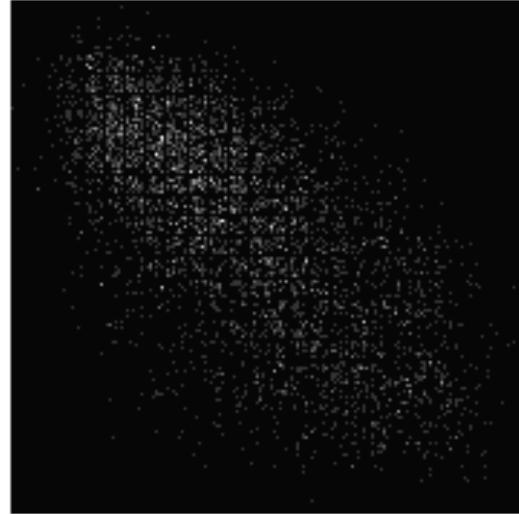
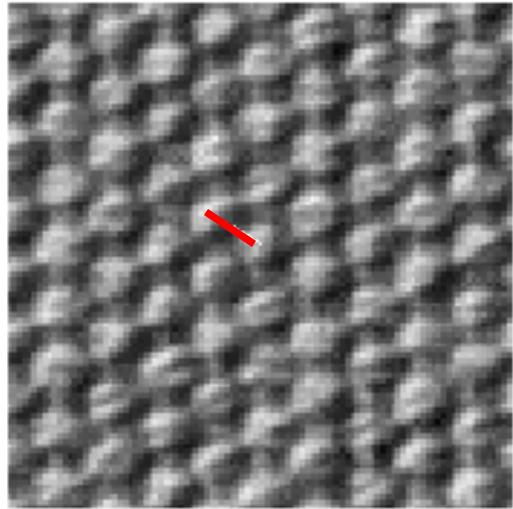
## Cooccurrence matrix

probability distributions for intensity pairs

Contains information on some aspects of the spatial configurations



## Cooccurrence matrix



## Cooccurrence matrix

Features calculated from the matrix:

feature	expression
energy	$\sum_i \sum_j C^2(i, j)$
entropy	$-\sum_i \sum_j C(i, j) \log C(i, j)$
contrast	$\sum_i \sum_j (i - j)^2 C(i, j)$
homogeneity	$\sum_i \sum_j C(i, j) / (1 +  i - j )$
max. probability	$\max_{i,j} C(i, j)$

## On the menu:

- Fourier features
- cooccurrence matrices
- filter banks (Laws, Gabor, eigenfilters)
- stochastic models)

# Computer Vision

Feature	1D filter	
L3	[ 1 2 1 ]	
E3	[ -1 0 1 ]	
S3	[ -1 2 -1 ]	
<hr/>		
L5	[ 1 4 6 4 1 ]	
E5	[ -1 -2 0 2 1 ]	
S5	[ -1 0 2 0 -1 ]	
W5	[ -1 2 0 -2 1 ]	
R5	[ 1 -4 6 -4 1 ]	
<hr/>		
L7	[ 1 6 15 20 15 6 1 ]	
E7	[ -1 -4 -5 0 5 4 1 ]	
S7	[ -1 -2 1 4 1 -2 -1 ]	
W7	[ -1 0 3 0 -3 0 1 ]	
R7	[ 1 -2 -1 4 -1 -2 1 ]	
O7	[ -1 6 -15 20 -15 6 -1 ]	
<hr/>		

## Laws filters

This fixed filter set yields simple convolutions but has proven very effective in some cases

## Gabor filters

Gaussian envelope multiplied by cosine

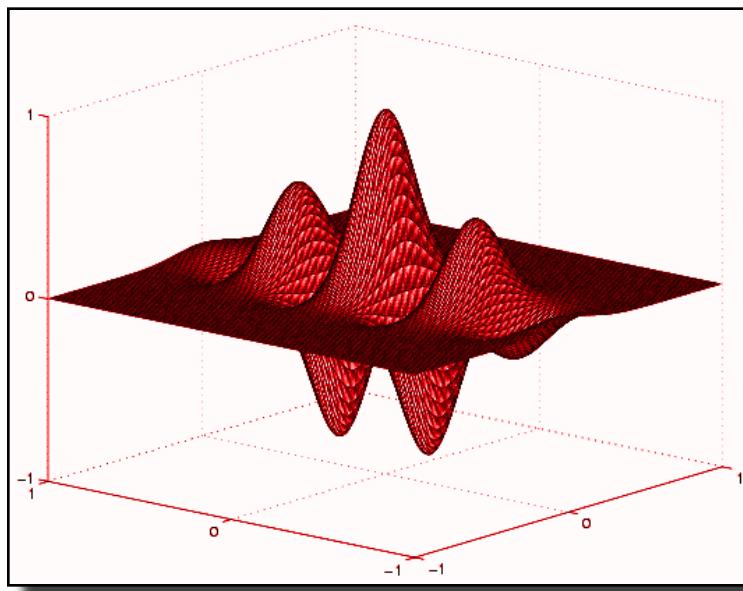
$$g(x, y) = e^{-\frac{x^2 + y^2}{4\Delta_{x,y}^2}} \cos(2\pi u^* x + \varphi)$$

The filter's Fourier power spectrum

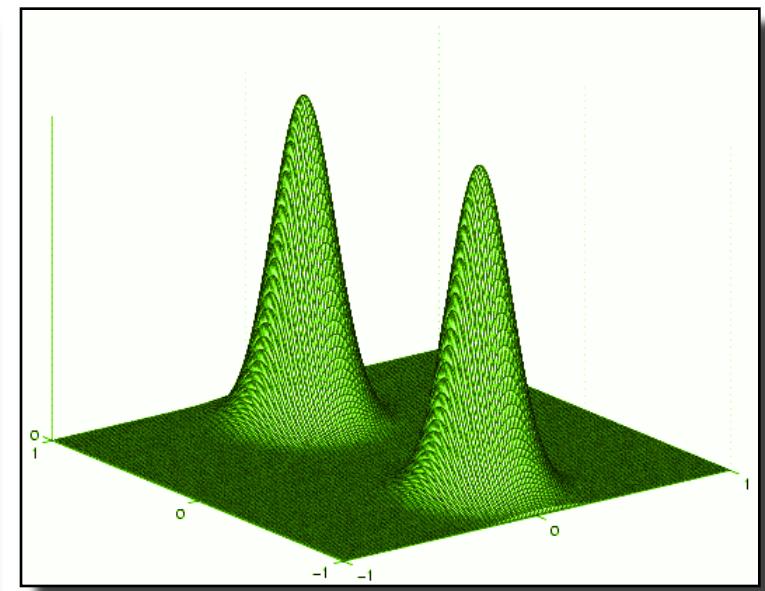
$$G(u, v) = \frac{1}{4\pi\Delta_{u,v}^2} \left( e^{-((u-u^*)^2+v^2)/(4\Delta_{u,v}^2)} + e^{-((u+u^*)^2+v^2)/(4\Delta_{u,v}^2)} \right)$$

# Gabor filters

Spatial domain



Frequency domain



**Good localisation in both domains**

# Gabor filters

$$f = f(x, y)$$

$$F = F(u, v)$$

$$x_{av} = \frac{\int_{-\infty}^{\infty} x f \bar{f} dx}{\int_{-\infty}^{\infty} f \bar{f} dx}$$

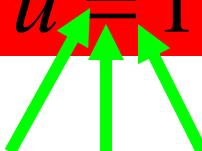
$$u_{av} = \frac{\int_{-\infty}^{\infty} u F \bar{F} du}{\int_{-\infty}^{\infty} F \bar{F} du}$$

$$(\Delta x)^2 = \frac{\int_{-\infty}^{\infty} (x - x_{av})^2 f \bar{f} dx}{\int_{-\infty}^{\infty} f \bar{f} dx}$$

$$(\Delta u)^2 = \frac{\int_{-\infty}^{\infty} (u - u_{av})^2 F \bar{F} du}{\int_{-\infty}^{\infty} F \bar{F} du}$$

**the Heisenberg uncertainty principle**

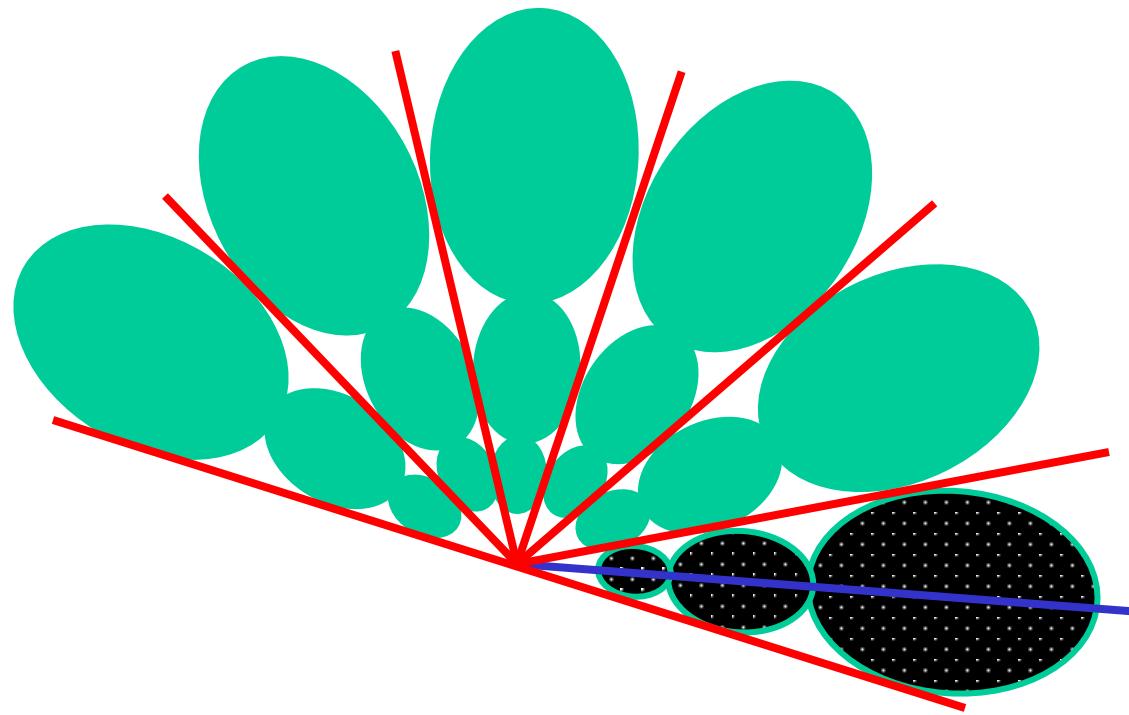
$$\Delta x \Delta u = 1/4\pi$$



## Gabor filters

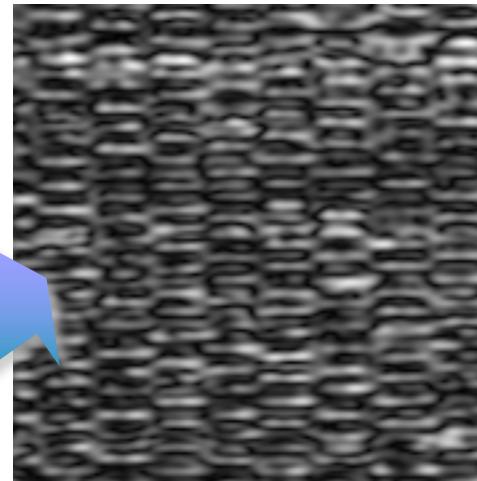
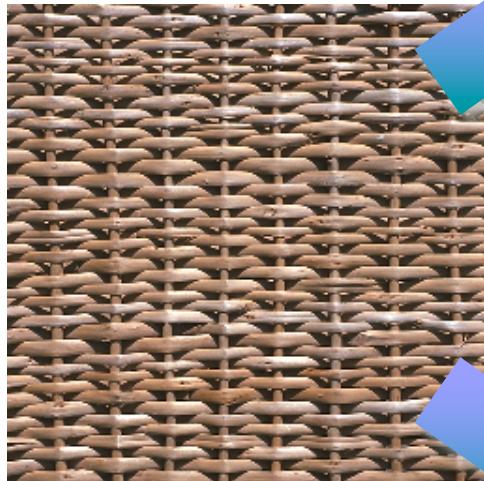
Covering the Fourier domain with responses

- to probe for directionality
- to look at different scales

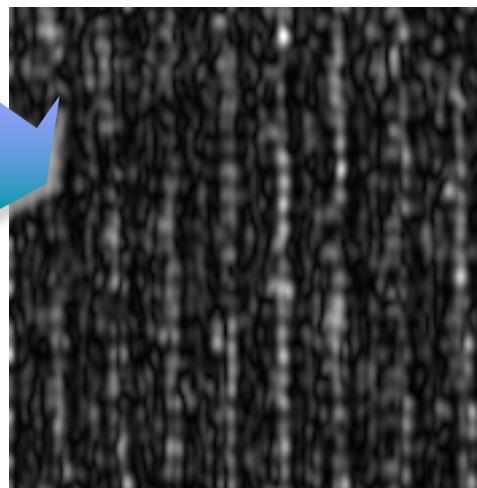


# Gabor filters

Input texture



Output for filter  
responsive to  
horizontal  
structures



Output for filter  
responsive to  
vertical  
structures

## Eigenfilters (Ade, ETH)

Filters adapted to the texture

- 1) shift mask over training image
- 2) collect intensity statistics
- 3) PCA -> eigenvectors -> 'eigenfilters'
- 4) energies of eigenfilter outputs

## Eigenfilters

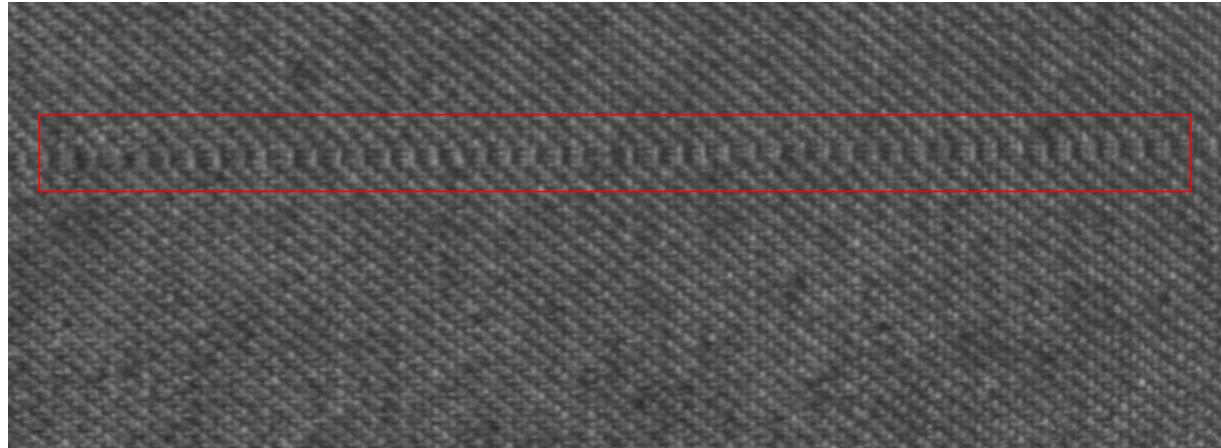
Filters adapted to the texture

but small filters may reduce efficacy

hence large, but sparse filters

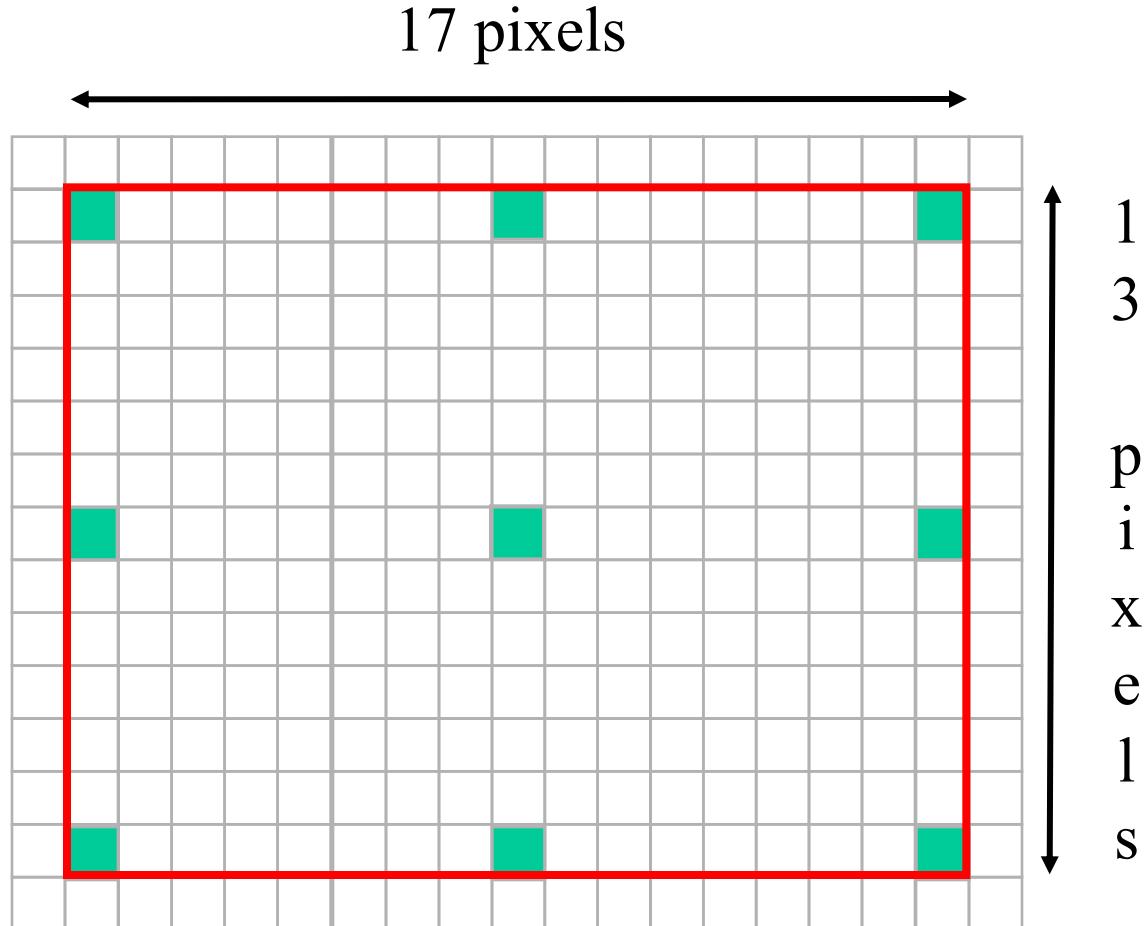
# Eigenfilters

Example applications: textile inspection



Filters with size of one period  
(period found as peak in autocorrelation)

# Eigenfilters



# Eigenfilters

## Covariance matrix needed for PCA

174.1	-77.6	101.6	-60.8	72.7	-71.5	116.5	-77.9	91.4
-77.6	173.9	-78.2	71.5	-61.7	73.1	-76.4	116.4	-78.4
101.6	-78.2	173.5	-70.4	71.7	-62.1	95.3	-77.0	116.3
-60.8	71.5	-70.4	173.9	-76.5	101.0	-59.4	71.8	-70.1
72.7	-61.7	71.7	-76.5	173.7	-77.1	70.6	-60.3	72.1
-71.5	73.1	-62.1	101.0	-77.1	173.4	-69.3	70.9	-60.7
116.5	-76.4	95.3	-59.4	70.6	-69.3	173.4	-75.3	99.8
-77.9	116.4	-77.0	71.8	-60.3	70.9	-75.3	173.2	-75.9
91.4	-78.4	116.3	-70.1	72.1	-60.7	99.8	-75.9	172.8

# Computer Vision

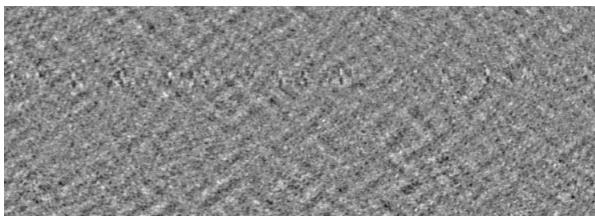
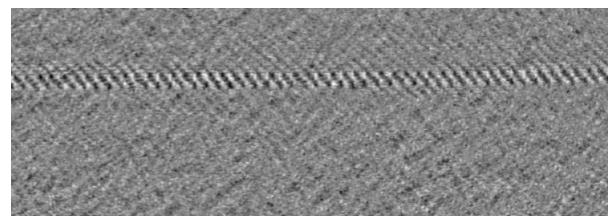
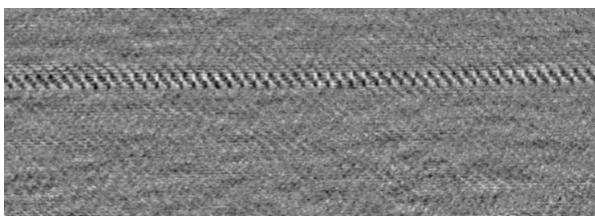
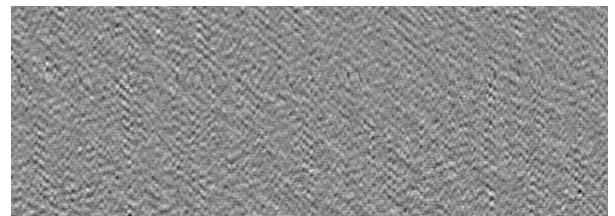
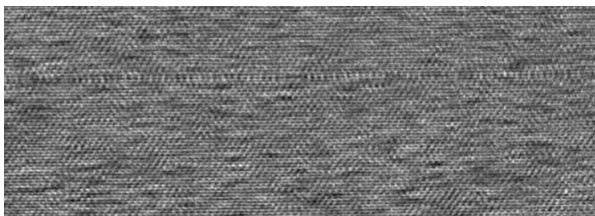
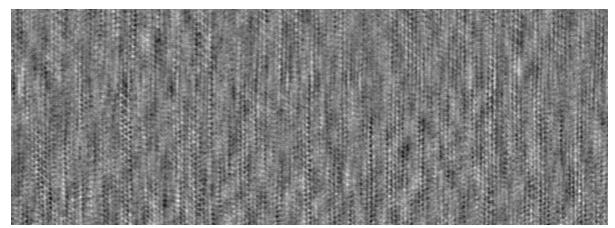
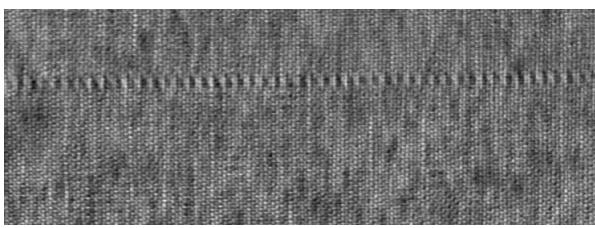
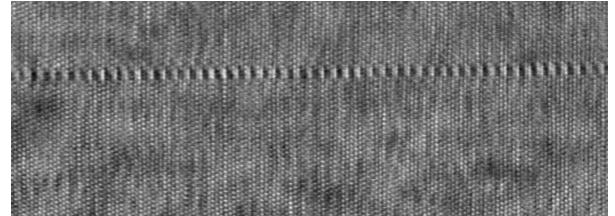
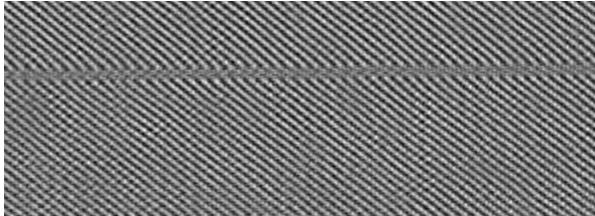
# Eigenfilters

$$\begin{pmatrix} 0.35 \\ -0.33 \\ 0.35 \\ -0.30 \\ 0.30 \\ -0.30 \\ 0.35 \\ -0.33 \\ 0.35 \end{pmatrix}, \begin{pmatrix} 0.31 \\ 0.12 \\ 0.31 \\ 0.51 \\ -0.20 \\ 0.49 \\ 0.34 \\ 0.12 \\ 0.31 \end{pmatrix}, \begin{pmatrix} 0.10 \\ 0.58 \\ 0.10 \\ -0.19 \\ 0.41 \\ -0.19 \\ 0.11 \\ 0.59 \\ 0.09 \end{pmatrix}, \begin{pmatrix} 0.46 \\ -0.00 \\ -0.43 \\ 0.30 \\ 0.03 \\ -0.28 \\ 0.41 \\ -0.02 \\ -0.49 \end{pmatrix}, \begin{pmatrix} -0.10 \\ -0.15 \\ -0.09 \\ 0.33 \\ 0.83 \\ 0.33 \\ -0.13 \\ -0.14 \\ -0.06 \end{pmatrix}$$

$$\begin{pmatrix} 0.27 \\ 0.11 \\ -0.11 \\ -0.62 \\ 0.00 \\ 0.63 \\ 0.11 \\ -0.12 \\ -0.27 \end{pmatrix}, \begin{pmatrix} -0.43 \\ -0.06 \\ -0.54 \\ -0.10 \\ 0.00 \\ 0.09 \\ 0.55 \\ 0.05 \\ 0.40 \end{pmatrix}, \begin{pmatrix} 0.08 \\ -0.69 \\ 0.02 \\ -0.09 \\ -0.00 \\ 0.10 \\ -0.02 \\ 0.69 \\ -0.09 \end{pmatrix}, \begin{pmatrix} -0.50 \\ -0.00 \\ 0.50 \\ 0.00 \\ 0.02 \\ 0.00 \\ 0.47 \\ -0.01 \\ -0.50 \end{pmatrix}$$

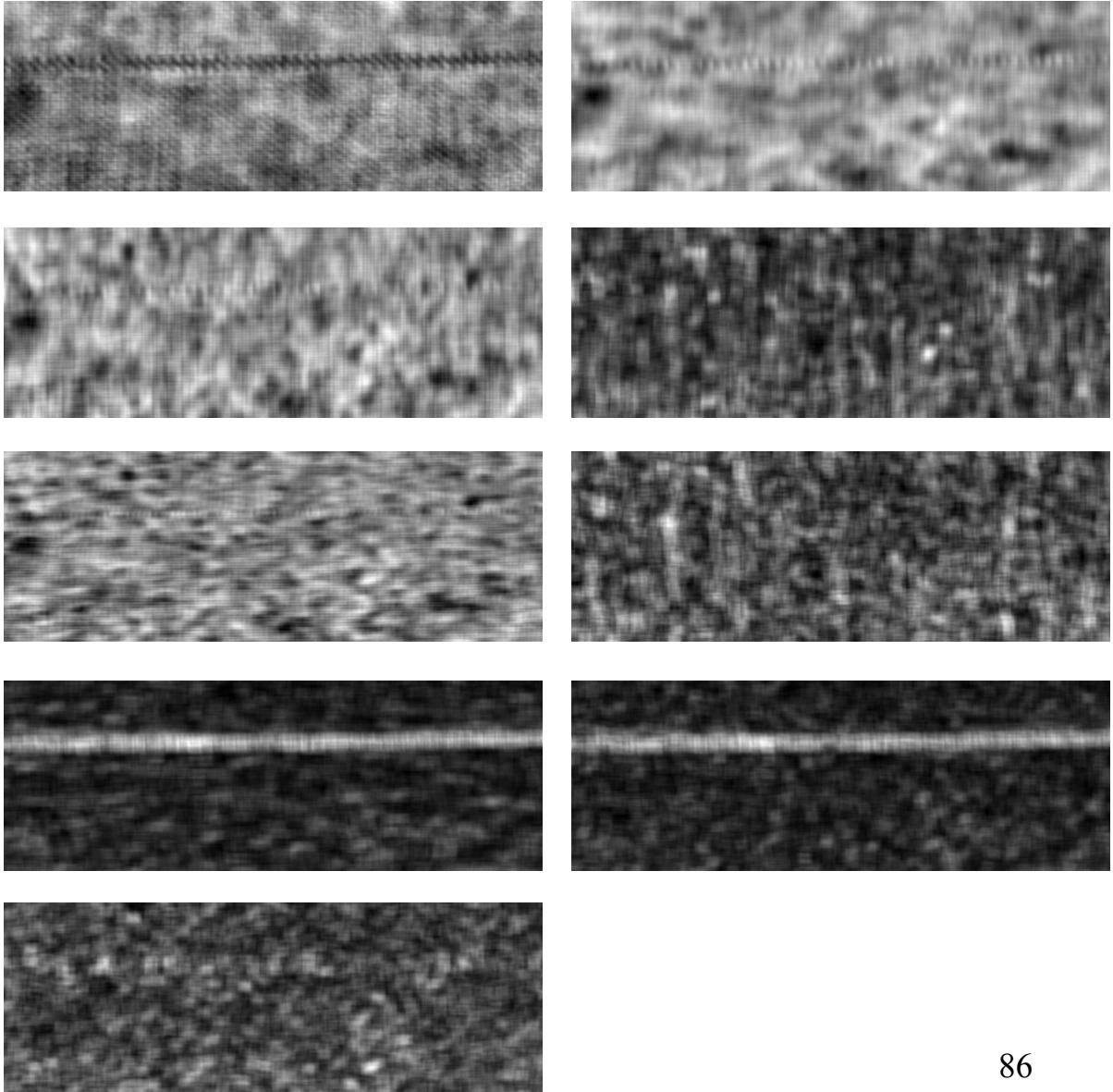
# Eigenfilters

e  
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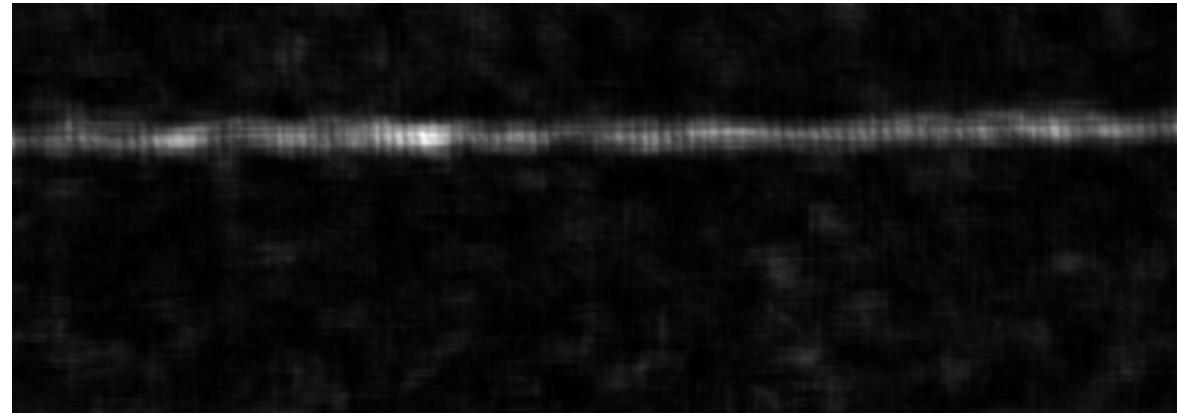
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# Eigenfilters

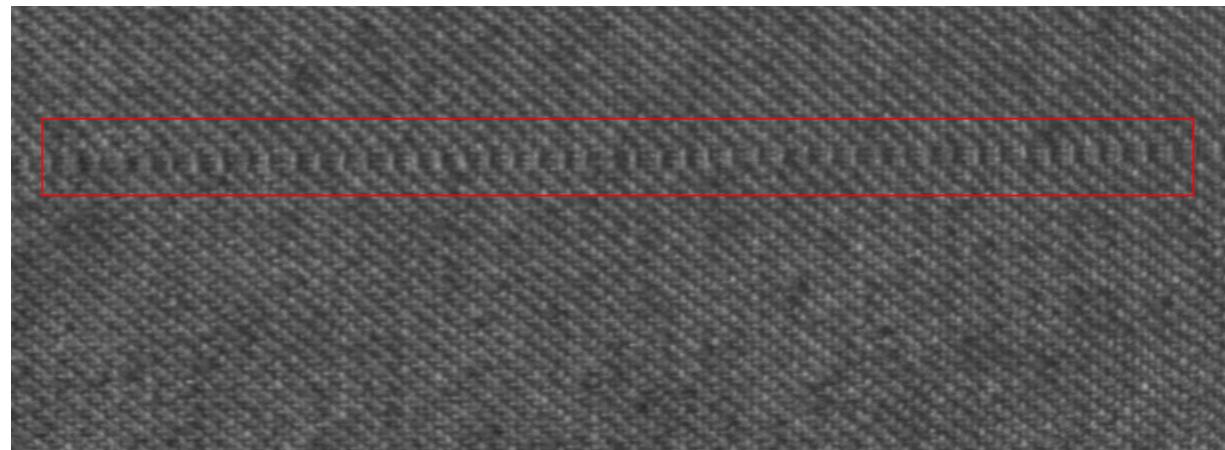


# Eigenfilters

Mahalanobis distance of the energies:

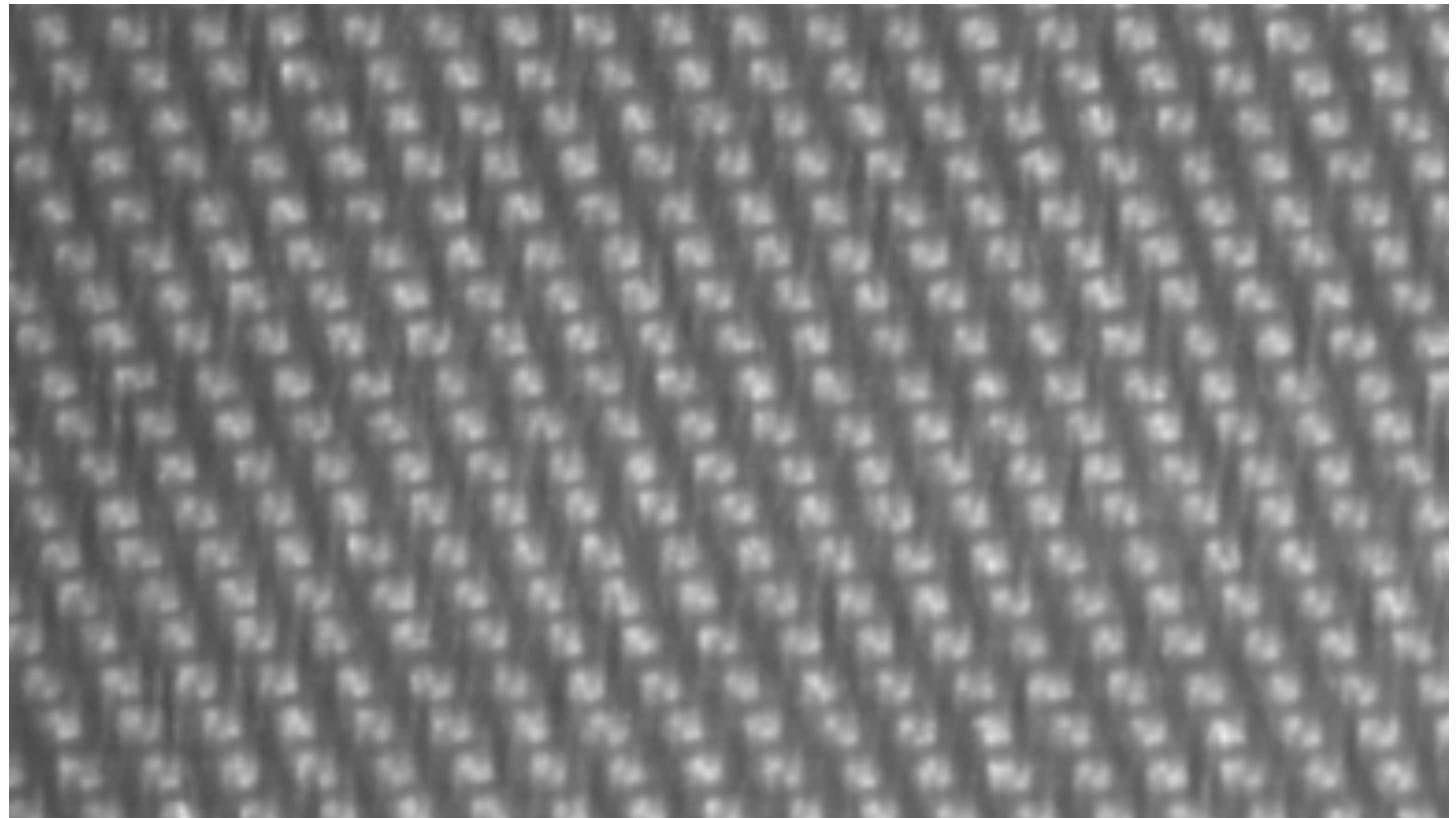


Flaw region found by thresholding:



# Eigenfilters

Textile inspection: a second example



The texture is coarser, the filters are larger...

# Eigenfilters

Textile inspection: eigenfilter blueprint

1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1

21 columns

47 rows

# Eigenfilters

## The covariance matrix

Covariance Matrix :

743,89	-331,82	632,59	-316,03	618,47	-298,31	632,41	-302,50	548,84
-331,82	741,71	-345,42	641,76	-330,40	614,36	-334,25	629,22	-316,68
632,59	-345,42	738,28	-338,50	638,50	-343,03	618,37	-347,55	624,75
-316,03	641,76	-338,50	746,54	-328,93	634,29	-314,59	619,83	-297,11
618,47	-330,40	638,50	-328,93	743,82	-343,16	643,10	-329,00	615,32
-298,31	614,36	-343,03	634,29	-343,16	739,56	-337,44	639,40	-342,03
632,41	-334,25	618,37	-314,59	643,10	-337,44	748,53	-328,18	636,26
-302,50	629,22	-347,55	619,83	-329,00	639,40	-328,18	745,57	-342,19
548,84	-316,68	624,75	-297,11	615,32	-342,03	636,26	-342,19	741,04

# Eigenfilters

## Eigenvectors / eigenvalues

Eigenvalues :

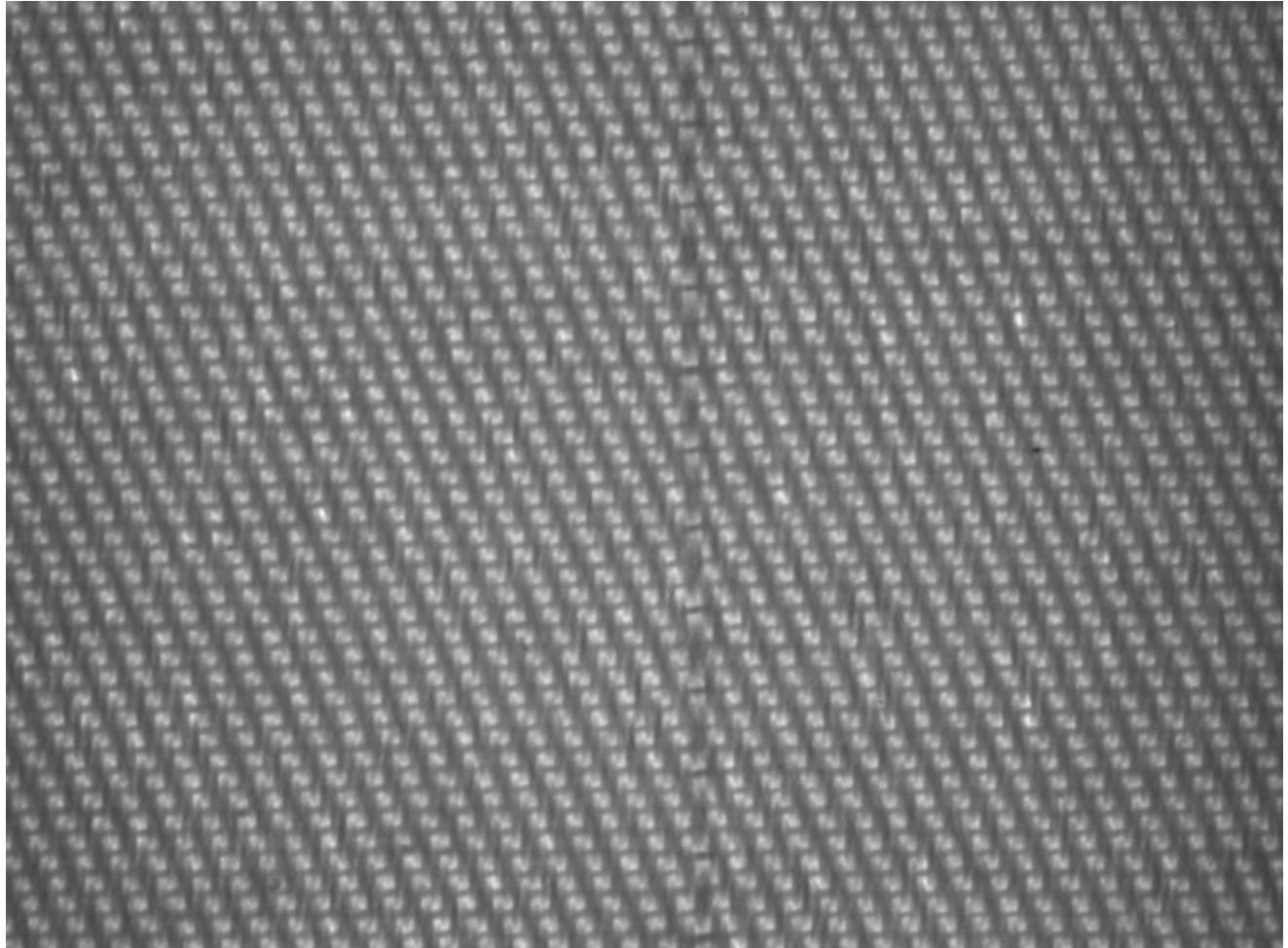
4428,70	1432,36	215,64	126,57	126,19	113,86	98,41	85,21	62,01
---------	---------	--------	--------	--------	--------	-------	-------	-------

Eigenvectors :

0,337	0,292	-0,621	-0,121	0,119	-0,319	-0,229	0,040	-0,480
-0,317	0,382	0,225	0,273	-0,369	-0,465	-0,302	-0,429	-0,025
0,353	0,271	-0,055	-0,434	-0,536	0,069	-0,179	0,155	0,513
-0,314	0,400	0,240	-0,294	0,259	-0,399	0,266	0,548	0,034
0,350	0,285	-0,005	0,404	-0,292	-0,005	0,734	-0,004	-0,096
-0,318	0,381	-0,234	-0,410	0,161	0,353	0,295	-0,541	0,033
0,350	0,294	0,056	0,312	0,610	-0,056	-0,144	-0,163	0,518
-0,317	0,386	-0,219	0,421	-0,081	0,530	-0,261	0,412	-0,025
0,340	0,274	0,630	-0,173	0,079	0,330	-0,198	-0,046	-0,475

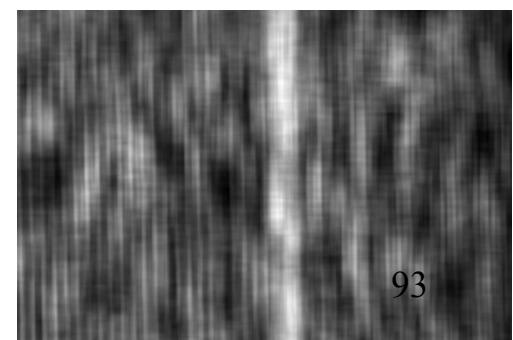
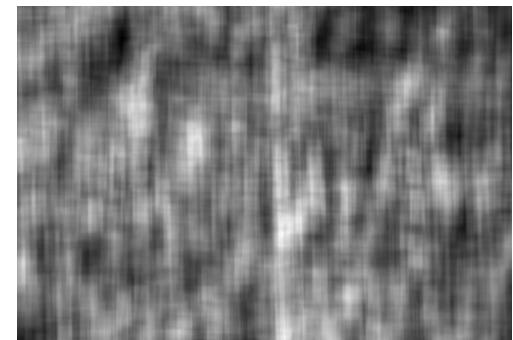
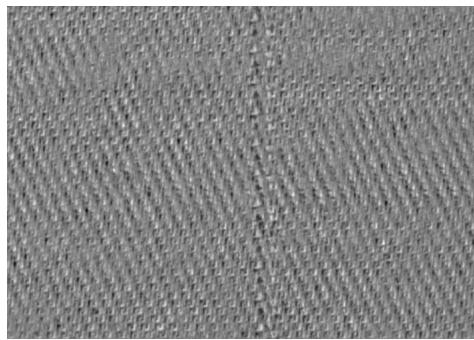
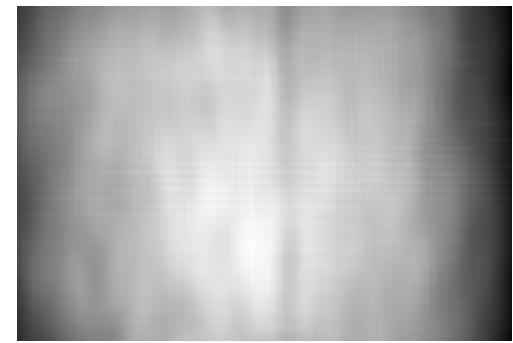
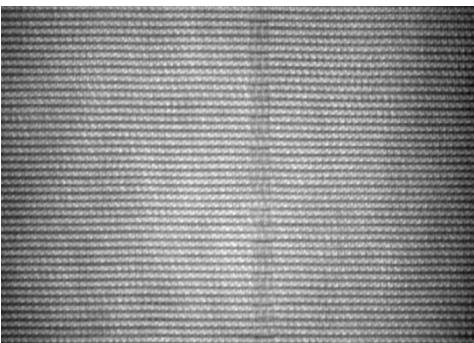
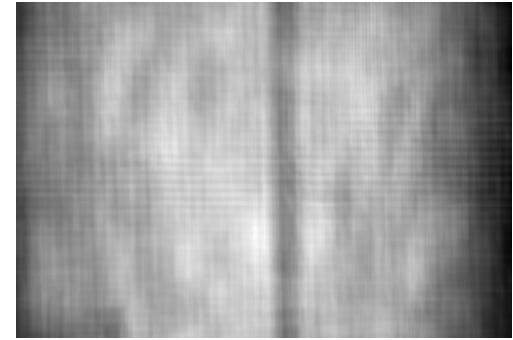
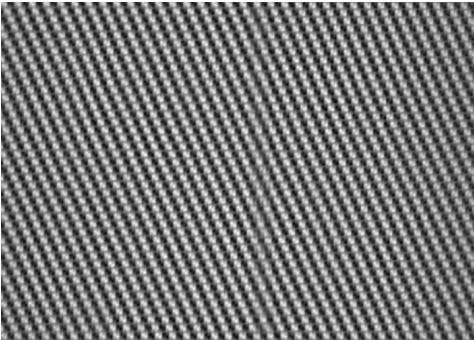
# Eigenfilters

example of defect



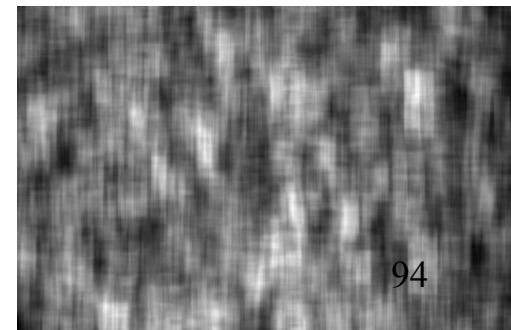
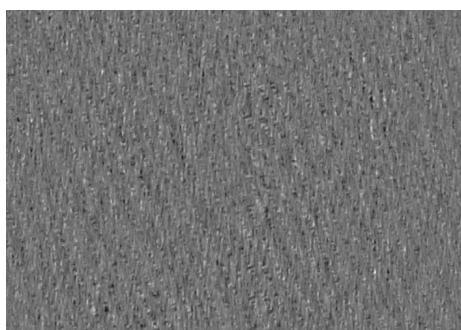
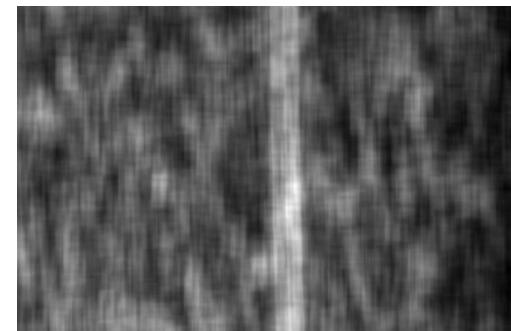
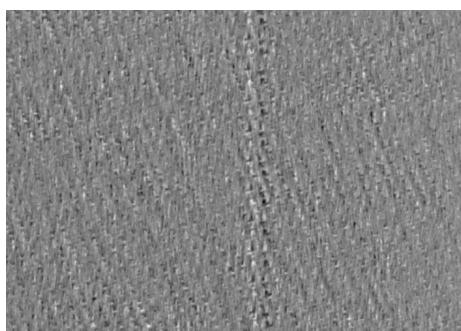
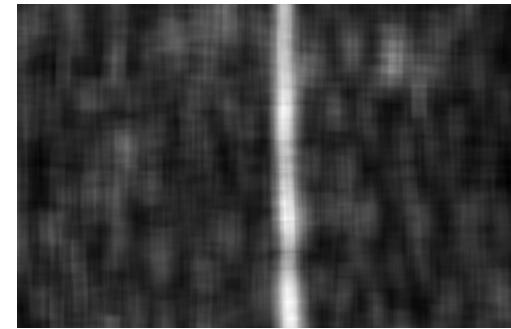
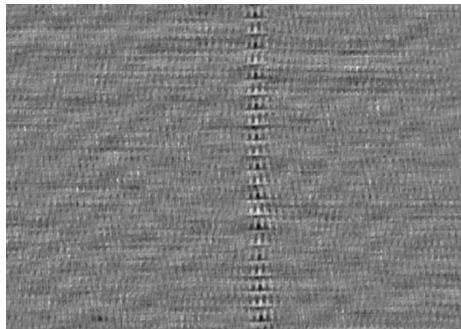
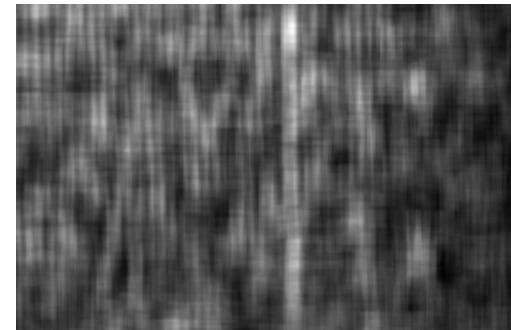
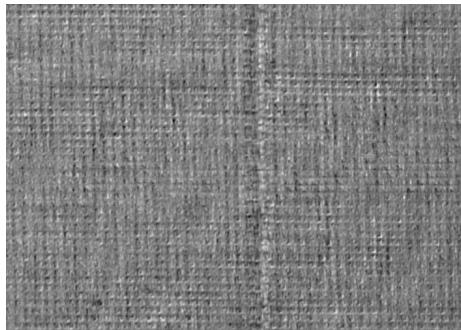
# Eigenfilters

Outputs/energies  
for the  
4 largest  
eigenvalues



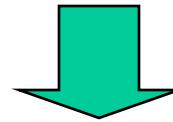
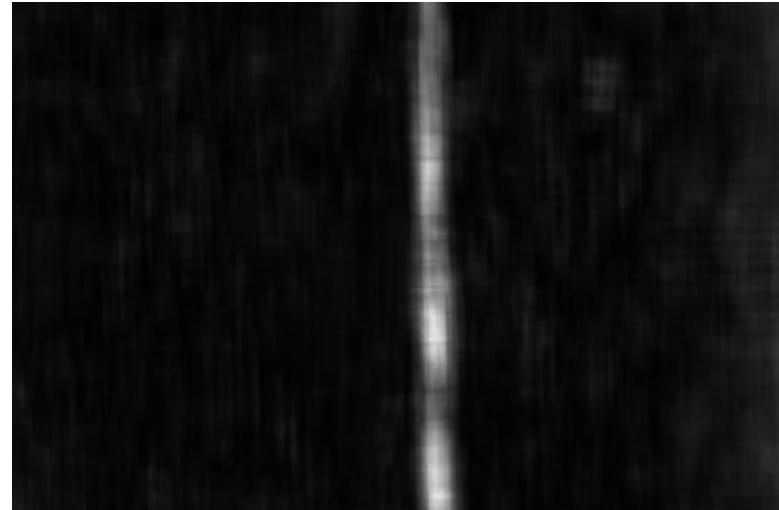
# Eigenfilters

4 smallest  
eigenvalues



# Eigenfilters

Mahalanobis  
distance



Threshold