Recording of Images
Acquisition of images

We focus on:

1. cameras
2. illumination
Acquisition of images

We focus on:

1. cameras
2. illumination
Acquisition of images

We focus on:

1. cameras
2. illumination
cameras
Optics for image formation

the pinhole model:
Optics for image formation

the pinhole model:

hence the name: **CAMERA obscura**
Optics for image formation

the pinhole model:

\[
\frac{X_i}{X_o} = \frac{Y_i}{Y_o} = \frac{f}{-Z_o} = -m
\]

\((m = \text{linear magnification})\)
Camera obscura + lens
The thin-lens equation

Lens to capture enough light:

\[
\frac{1}{Z_O} - \frac{1}{Z_i} = \frac{1}{f}
\]

assuming
- spherical lens surfaces
- incoming light ± parallel to axis
- thickness \(<<\) radii
- same refractive index on both sides
The depth-of-field

\[ \Delta Z_0^- = Z_0 - Z_0^- = \frac{Z_0 (Z_0 - f)}{Z_0 + f \frac{d}{b - f}} \]

decreases with \( d \), increases with \( Z_0 \)

strike a balance between incoming light and large depth-of-field (usable depth range)
The depth-of-field

\[ \Delta Z^+_0 = Z_0 - Z^-_0 = \frac{Z_0 (Z_0 - f)}{Z_0 + f \frac{d}{b} - f} \]

Ex 1: microscopes -> small DoF

Ex 2: special effects -> flood miniature scene with light
Deviations from the lens model

3 assumptions:

1. all rays from a point are focused onto 1 image point

2. all image points in a single plane

3. magnification is constant

deviations from this ideal are *aberrations*
Aberrations

2 types:

1. geometrical
2. chromatic

*geometrical*: small for paraxial rays

*chromatic*: refractive index function of wavelength (Snell’s law !!)
Geometrical aberrations

- spherical aberration
- astigmatism
- radial distortion
- coma

the most important type
Radial Distortion

magnification different for different angles of inclination

barrel

none

pincushion
Radial Distortion

magnification different for different angles of inclination

barrel  none  pincushion

The result is pixels moving along lines through the center of the distortion – typically close to the image center – over a distance \( d \), depending on the pixels’ distance \( r \) to the center:

\[
d = (1 + \kappa_1 r^2 + \kappa_2 r^4 + \ldots)
\]
Radial Distortion

magnification different for different angles of inclination

This aberration type can be corrected by software if the parameters \((\kappa_1, \kappa_2, \ldots)\) are known.
Radial Distortion

magnification different for different angles of inclination

Some methods do this by looking how lines curve instead of being straight
Chromatic aberration

rays of different wavelengths focused in different planes cannot be removed completely but **achromatization** can be achieved at some well chosen wavelength pair, by combining lenses made of different glasses

sometimes **achromatization** is achieved for more than 2 wavelengths
Cameras

we consider 2 types:

1. CCD
2. CMOS
**CCD vs. CMOS**

- Niche applications
- Specific technology
- High production cost
- High power consumption
- Higher fill rate
- Blooming
- Sequential readout

- Consumer cameras
- Standard IC technology
- Cheap
- Low power
- Less sensitive
- Per pixel amplification
- Random pixel access
- Smart pixels
- On chip integration with other components

Disadvantages of CMOS largely lifted by now
CCD vs. CMOS

- Niche applications
- Specific technology
- High production cost
- High power consumption
- Higher fill rate
- Blooming
- Sequential readout

- Consumer cameras
- Standard IC technology
- Cheap
- Low power
- Less sensitive
- Per pixel amplification
- Random pixel access
- Smart pixels
- On chip integration with other components

2006 was year of sales cross-over: more CMOS
CCD vs. CMOS

- Niche applications
- Specific technology
- High production cost
- High power consumption
- Higher fill rate
- Blooming
- Sequential readout

- Consumer cameras
- Standard IC technology
- Cheap
- Low power
- Less sensitive
- Per pixel amplification
- Random pixel access
- Smart pixels
- On chip integration with other components

In 2015 Sony said to stop CCD chip production
Resolution trend in mobile phones

*Volume and revenue opportunity for high resolution sensors*

Source: TSR, CCD/CMOS Area Image Sensor Market Analysis, dated June 2011
Colour cameras

• We consider 3 concepts:

1. Prism (with 3 sensors)
2. Filter mosaic
3. Filter wheel
Prism colour camera

Separate light in 3 beams using dichroic prism
Requires 3 sensors & precise alignment
Good color separation
Prism colour camera
Filter mosaic

Coat filter directly on sensor

Demosaicing (obtain full colour & full resolution image)
Color filters lower the effective resolution, hence microlenses often added to gain more light on the small pixels.
Filter wheel

Rotate multiple filters in front of lens
Allows more than 3 colour bands

Only suitable for static scenes
<table>
<thead>
<tr>
<th>Approach</th>
<th>Prism</th>
<th>Mosaic</th>
<th>Wheel</th>
</tr>
</thead>
<tbody>
<tr>
<td># sensors</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Resolution</td>
<td>High</td>
<td>Average</td>
<td>Good</td>
</tr>
<tr>
<td>Cost</td>
<td>High</td>
<td>Low</td>
<td>Average</td>
</tr>
<tr>
<td>Frame rate</td>
<td>High</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Artefacts</td>
<td>Low</td>
<td>Aliasing</td>
<td>Motion</td>
</tr>
<tr>
<td>Bands</td>
<td>3</td>
<td>3</td>
<td>3 or more</td>
</tr>
<tr>
<td></td>
<td>High-end cameras</td>
<td>Low-end cameras</td>
<td>Scientific applications</td>
</tr>
</tbody>
</table>
Geometric camera model
perspective projection

(Man Drawing a Lute, woodcut, 1525, Albrecht Dürer)
Models for camera projection

the pinhole model revisited:

center of the lens = center of projection

notice the virtual image plane

this is called **perspective** projection
Models for camera projection

We had the virtual plane also in the original reference sketch:
Perspective projection

principal point

center of projection

(u, v)

(Xc, Yc, Zc)

image plane

Xc

Yc

Zc

f

u

v
Perspective projection

- origin lies at the center of projection
- the $Z_c$ axis coincides with the optical axis
- $X_c$-axis $\parallel$ to image rows, $Y_c$-axis $\parallel$ to columns
Computer Vision

Perspective projection

\[ \frac{u}{X} = \frac{f}{Z} \quad \frac{v}{Y} = \frac{f}{Z} \]
Perspective projection

\[ u = f \frac{X}{Z} \]

\[ v = f \frac{Y}{Z} \]
Pseudo-orthographic projection

\[ u = f \frac{X}{Z} \quad \quad v = f \frac{Y}{Z} \]

If \( Z \) is constant \( \Rightarrow x = kX \) and \( y = kY \), where \( k = \frac{f}{Z} \)

i.e. orthographic projection + a scaling

Good approximation if \( \frac{f}{Z} \) is constant, i.e. if objects are small compared to their distance from the camera.
Pictoral comparison

Pseudo-orthographic

Perspective
Projection matrices

the perspective projection model is incomplete:
what if:

1. 3D coordinates are specified in a
   *world coordinate frame*

2. Image coordinates are expressed as
   *row and column numbers*

We will not consider additional refinements, such as radial distortions,...
Projection matrices

\[ u = f \frac{\langle r_1, P - C \rangle}{\langle r_3, P - C \rangle} \]

\[ v = f \frac{\langle r_2, P - C \rangle}{\langle r_3, P - C \rangle} \]

\[ u = f \frac{\sum_{i=1}^{3} r_{1i}(X - C_i)}{\sum_{i=1}^{3} r_{3i}(X - C_i)} \]

\[ v = f \frac{\sum_{i=1}^{3} r_{2i}(Y - C_i)}{\sum_{i=1}^{3} r_{3i}(X - C_i)} \]
Projection matrices

Image coordinates are to be expressed as **pixel coordinates**

\[
\begin{align*}
x &= k_x u + s v + x_0 \\
y &= k_y v + y_0
\end{align*}
\]

with:

\[\rightarrow (x_0, y_0)\] the pixel coordinates of the principal point

\[\rightarrow NB7 : fully calibrated\] means internally and externally calibrated

\[\rightarrow s\] indicates the skew, typically \(s = 0\)
Homogeneous coordinates

Often used to linearize non-linear relations

2D
\[
\begin{pmatrix}
    x \\
    y \\
    z
\end{pmatrix}
\rightarrow
\begin{pmatrix}
    x / z \\
    y / z
\end{pmatrix}
\]

3D
\[
\begin{pmatrix}
    X \\
    Y \\
    Z \\
    W
\end{pmatrix}
\rightarrow
\begin{pmatrix}
    X / W \\
    Y / W \\
    Z / W
\end{pmatrix}
\]

Homogeneous coordinates are only defined up to a factor.
Projection matrices

\[ u = f \frac{r_{11}(X - C_1) + r_{12}(Y - C_2) + r_{13}(Z - C_3)}{r_{31}(X - C_1) + r_{32}(Y - C_2) + r_{33}(Z - C_3)} \]

\[ v = f \frac{r_{21}(X - C_1) + r_{22}(Y - C_2) + r_{23}(Z - C_3)}{r_{31}(X - C_1) + r_{32}(Y - C_2) + r_{33}(Z - C_3)} \]

Exploiting homogeneous coordinates:

\[
\begin{pmatrix}
u \\ v \\ \tau \\
1
\end{pmatrix} =
\begin{pmatrix}
f & r_{11} & f & r_{12} & f & r_{13} \\
f & r_{21} & f & r_{22} & f & r_{23} \\
r_{31} & r_{32} & r_{33}
\end{pmatrix}
\begin{pmatrix}
X - C_1 \\
Y - C_2 \\
Z - C_3
\end{pmatrix}
\]
Projection matrices

\[
\begin{align*}
x &= k_x u + s v + x_0 \\
y &= k_y v + y_0
\end{align*}
\]

Exploiting homogeneous coordinates:

\[
\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} =
\begin{pmatrix} k_x & s & x_0 \\ 0 & k_y & y_0 \\ 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} \tau u \\ \tau v \\ 1 \end{pmatrix}
\]
Projection matrices

Thus, we have:

\[
\begin{pmatrix}
    u \\
    v \\
    1
\end{pmatrix}
= \begin{pmatrix}
    f & r_{11} & f & r_{12} & f & r_{13} \\
    f & r_{21} & f & r_{22} & f & r_{23} \\
    r_{31} & r_{32} & r_{33}
\end{pmatrix}
\begin{pmatrix}
    X - C_1 \\
    Y - C_2 \\
    Z - C_3
\end{pmatrix}
\]

\[
\begin{pmatrix}
    x \\
    y \\
    1
\end{pmatrix}
= \begin{pmatrix}
    k_x & s & x_0 \\
    0 & k_y & y_0 \\
    0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    u \\
    v \\
    1
\end{pmatrix}
\]
Projection matrices

Concatenating the results:

\[
\tau \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} k_x & s & x_0 \\ 0 & k_y & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & r_{11} & f & r_{12} & f & r_{13} \\ f & r_{21} & f & r_{22} & f & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} X - C_1 \\ Y - C_2 \\ Z - C_3 \end{pmatrix}
\]

Or, equivalently:

\[
\tau \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} k_x & s & x_0 \\ 0 & k_y & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} X - C_1 \\ Y - C_2 \\ Z - C_3 \end{pmatrix}
\]
Projection matrices

Re-combining matrices in the concatenation:

\[
\begin{pmatrix}
  x \\
  y \\
  1
\end{pmatrix}
= 
\begin{pmatrix}
  k_x & s & x_0 \\
  0 & k_y & y_0 \\
  0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  f & 0 & 0 \\
  0 & f & 0 \\
  0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  r_{11} & r_{12} & r_{13} \\
  r_{21} & r_{22} & r_{23} \\
  r_{31} & r_{32} & r_{33}
\end{pmatrix}
\begin{pmatrix}
  X - C_1 \\
  Y - C_2 \\
  Z - C_3
\end{pmatrix}
\]

yields the calibration matrix \( K \):

\[
K = 
\begin{pmatrix}
  k_x & s & x_0 \\
  0 & k_y & y_0 \\
  0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  f & 0 & 0 \\
  0 & f & 0 \\
  0 & 0 & 1
\end{pmatrix}
= 
\begin{pmatrix}
  f & k_x & f & s & x_0 \\
  0 & f & k_y & y_0 \\
  0 & 0 & 0 & 1
\end{pmatrix}
\]
Projection matrices

We define

\[
p = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}, \quad P = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \quad \tilde{P} = \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}
\]

yielding

\[
\rho p = KR^t(P - C)
\]

for some non-zero \( \rho \in \mathbb{R} \)

or,

\[
\rho p = K \left( R^t \mid -R^tC \right) \tilde{P}
\]

or,

\[
\rho p = (M \mid t) \tilde{P} \quad \text{with rank } M = 3\]
From object radiance to pixel grey levels

After the geometric camera model...

... a photometric camera model (i.e. not where a world patch projects in the image but how bright)

2 steps:
1. from object radiance to image irradiance
2. from image irradiance to pixel grey level
Image irradiance and object radiance

we look at the irradiance $I$ that an object patch will cause in the image

assumptions:
- radiance $R$ assumed known
- object at large distance compared to the focal length

Is image irradiance directly related to the radiance of the image patch?
The viewing conditions

\[ I = R \frac{A_l}{f^2} \cos^4 \alpha \]

the \( \cos^4 \) law
The viewing conditions

\[ I = R \frac{A_i}{f^2} \cos^4 \alpha \]

thus, images quickly get darker near the rim
The $\cos^4 \theta$ law cont’d

Especially strong effects for wide-angle and fisheye lenses
From irradiance to gray levels

\[ f = gI^\gamma + d \]
illumination
Illumination

Well-designed illumination often is key in visual inspection

The light was good, but the hot wax was a problem...
Illumination techniques

Simplify the image processing by controlling the environment

An overview of illumination techniques:

1. back-lighting
2. directional-lighting
3. diffuse-lighting
4. polarized-lighting
5. coloured-lighting
6. structured-lighting
7. stroboscopic lighting
Back-lighting

lamps placed behind a transmitting diffuser plate, light source behind the object

generates high-contrast silhouette images, easy to handle with *binary vision*

often used in inspection
Example backlighting
Directional and diffuse lighting

**Directional-lighting**

generate sharp shadows

generation of specular reflection

(e.g. crack detection)

shadows and shading yield information about shape

**Diffuse-lighting**

illuminates uniformly from all directions

prevents sharp shadows and large intensity variations over glossy surfaces
Crack detection
Example directional lighting
Example diffuse lighting
Polarized lighting

2 uses:

1. to improve contrast between Lambertian and specular reflections

2. to improve contrasts between dielectrics and metals
Polarised lighting

polarizer/analyzer configurations

law of Malus:

\[ I(\theta) = I(0) \cos^2 \theta \]
Polarized lighting

2 uses:

1. to improve contrast between Lambertian and specular reflections

2. to improve contrasts between dielectrics and metals
Polarized lighting

2 uses:

1. to improve contrast between Lambertian and specular reflections

2. to improve contrasts between dielectrics and metals
Polarized lighting

specular reflection keeps polarisation:
diffuse reflection depolarises

suppression of specular reflection:

polarizer/analyzer crossed
prevents the large dynamic range caused by glare
Example pol. lighting (pol./an.crossed)
Polarized lighting

2 uses:

1. to improve contrast between Lambertian and specular reflections

2. to improve contrasts between dielectrics and metals
Reflection : dielectric

Polarizer at *Brewster angle*
Reflection: conductor

strong reflectors
more or less preserve polarization
Polarised lighting

distinction between specular reflection from dielectrics and metals;
works under the Brewster angle for the dielectric
dielectric has no parallel comp. ; metal does
suppression of specular reflection from dielectrics :
polarizer/ analyzer aligned
distinguished metals and dielectrics
Example pol. lighting (pol./an. aligned)
Coloured lighting

highlight regions of a similar colour

with band-pass filter: only light from projected pattern (e.g. monochromatic light from a laser)

differentiation between specular and diffuse reflection

comparing colours $\Rightarrow$ same spectral composition of sources!

spectral sensitivity function of the sensors!
Computer Vision

Example coloured lighting
Coloured lighting

*Example videos*: weed-selective herbicide spraying
Coloured lighting
Structured and stroboscopic lighting

spatially or temporally modulated light pattern

**Structured lighting**

e.g. : 3D shape : objects distort the projected pattern  
(more on this later)

**Stroboscopic lighting**

high intensity light flash

to eliminate motion blur
Stroboscopic lighting
Application

*Example videos*: vegetable inspection
Application